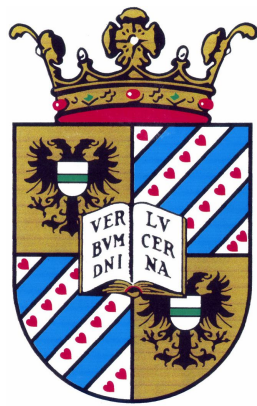


# Dynamic magnetoelectric coupling in Mott insulators



Maxim Mostovoy

*University of Groningen  
Zernike Institute  
for Advanced Materials*

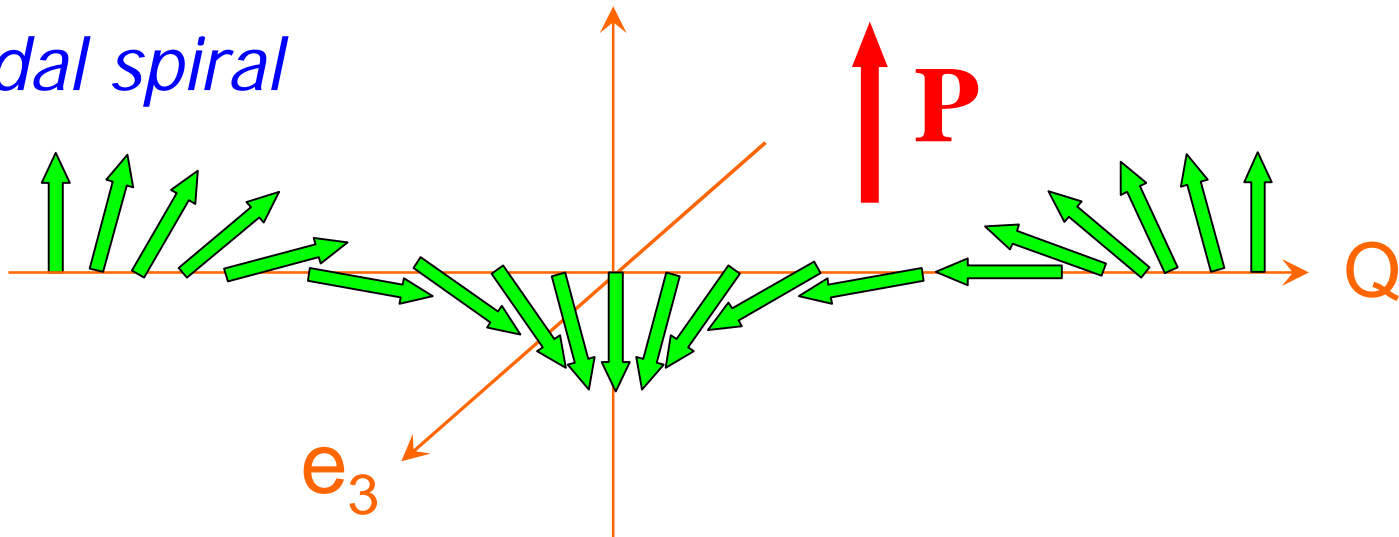
Modern Trends in Magnetism  
Grenoble  
September 9, 2011

# Outline

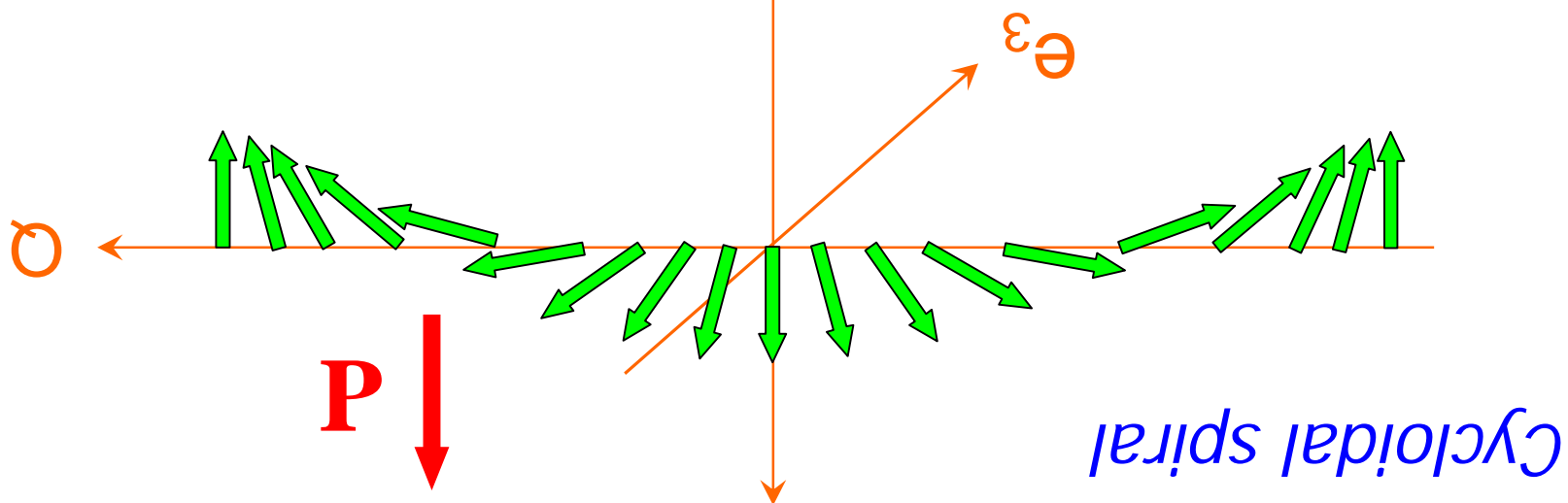
- **Dynamic magnetoelectric coupling**
- **Solitonic array and Yukawa forces in  $\text{TbFeO}_3$**

# Breaking of inversion symmetry by spin ordering

*Cycloidal spiral*



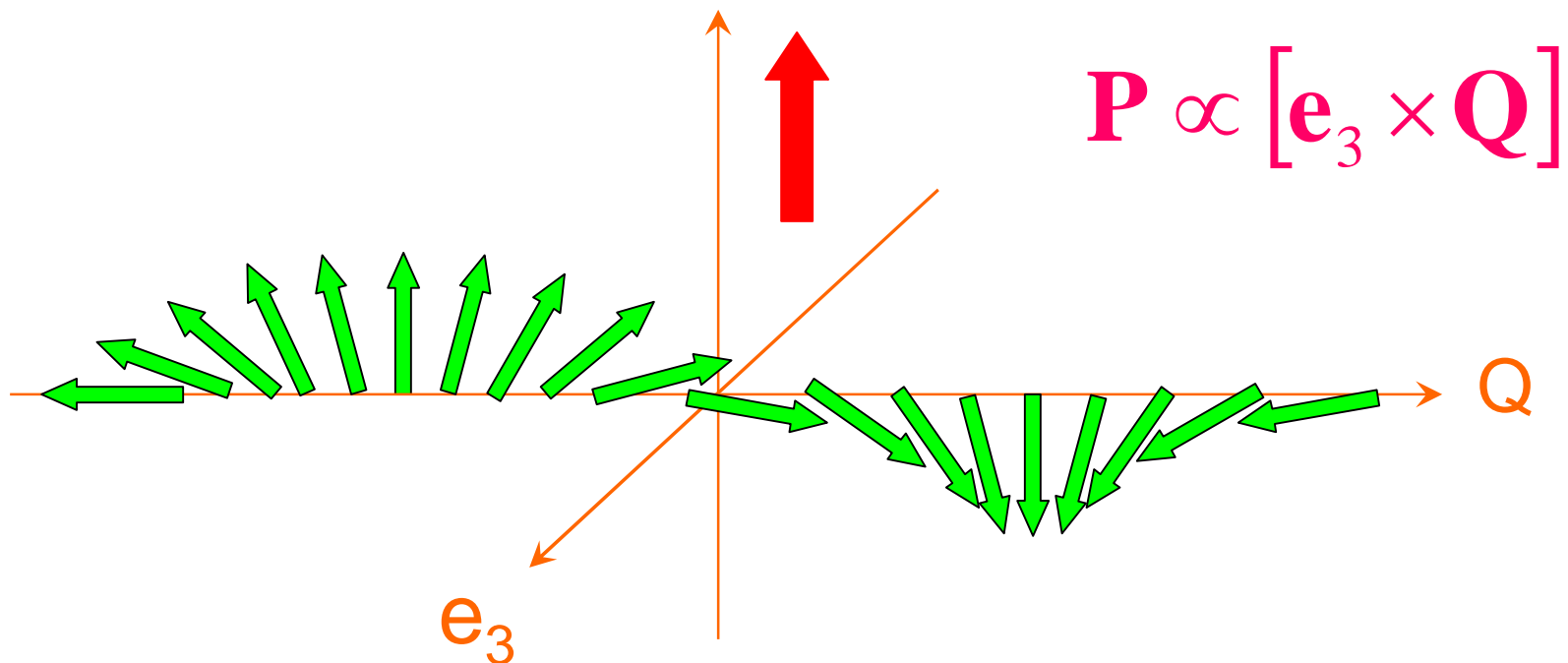
Inversion I:  $(x,y,z) \rightarrow (-x,-y,-z)$



# Induced Polarization

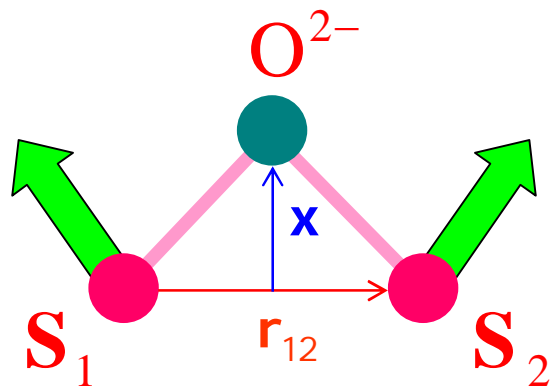
*H. Katsura et al PRL (2005), Sergienko & Dagotto PRB (2006), M.M. PRL (2006)*

$$P_y (M_x \partial_x M_y - M_y \partial_x M_x)$$



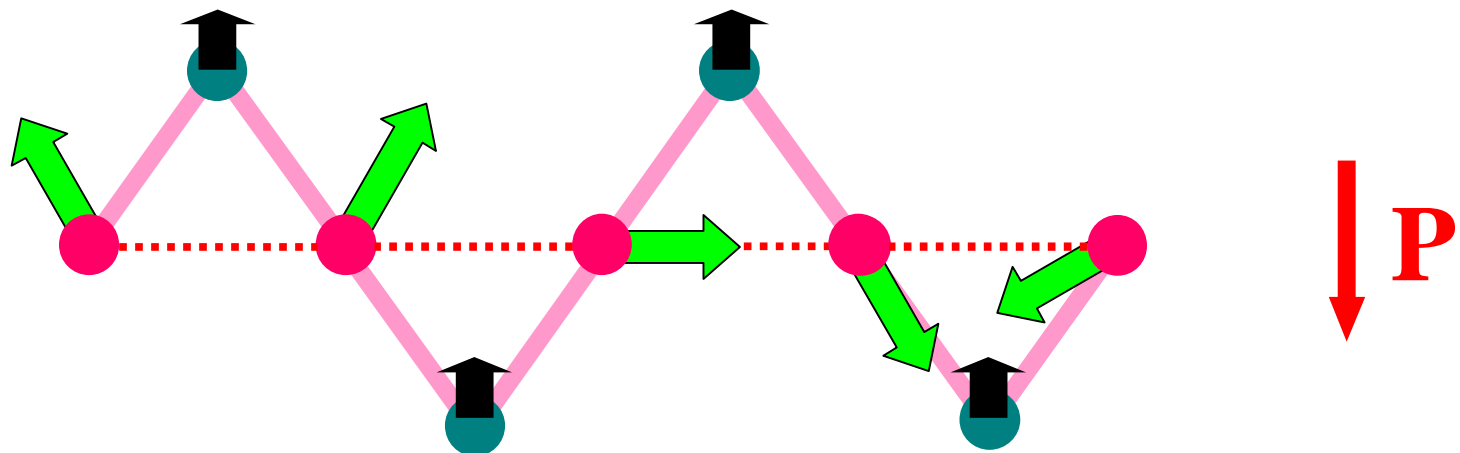
*I. Dzyaloshinskii, Theory of helical structures in antiferromagnets, Sov. Phys. JETP **19**, 960-971 (1964)*

# Inverse Dzyaloshinskii-Moriya mechanism



$$E_{DM} = \mathbf{D}_{12} \cdot [\mathbf{S}_1 \times \mathbf{S}_2]$$

$$\mathbf{D}_{12} \propto \lambda \mathbf{x} \times \hat{\mathbf{r}}_{12}$$



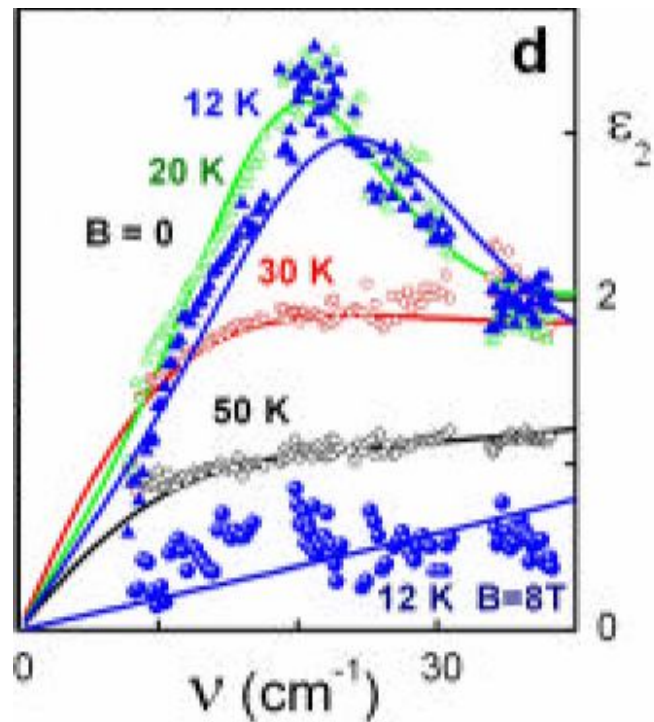


# Electromagnons

magnons excited by the  
electric field of light wave

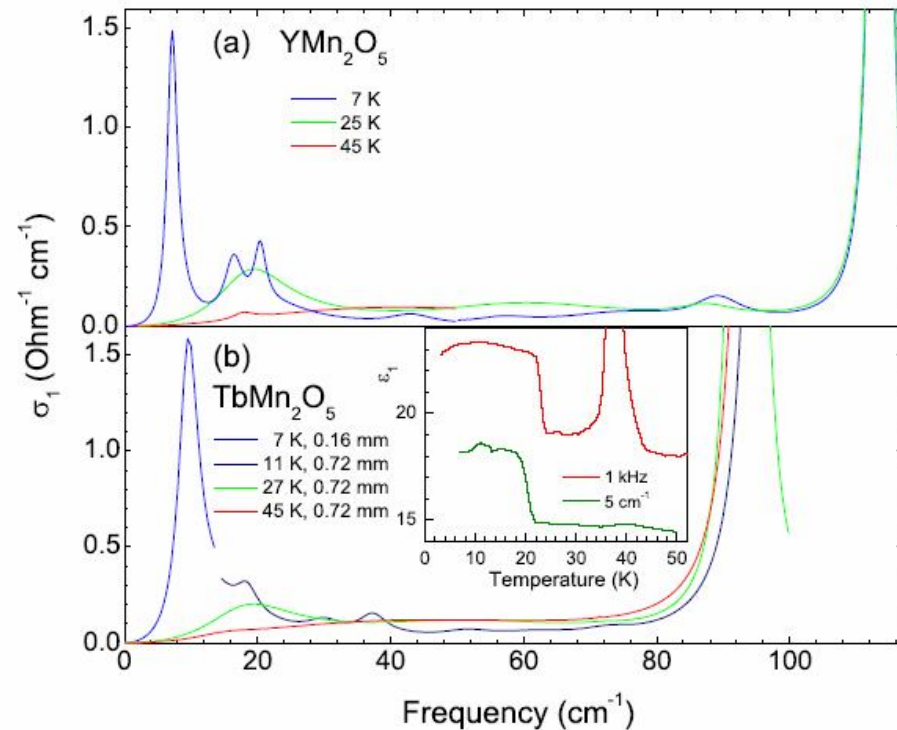
# Electromagnons in spiral multiferroics

TbMnO<sub>3</sub>



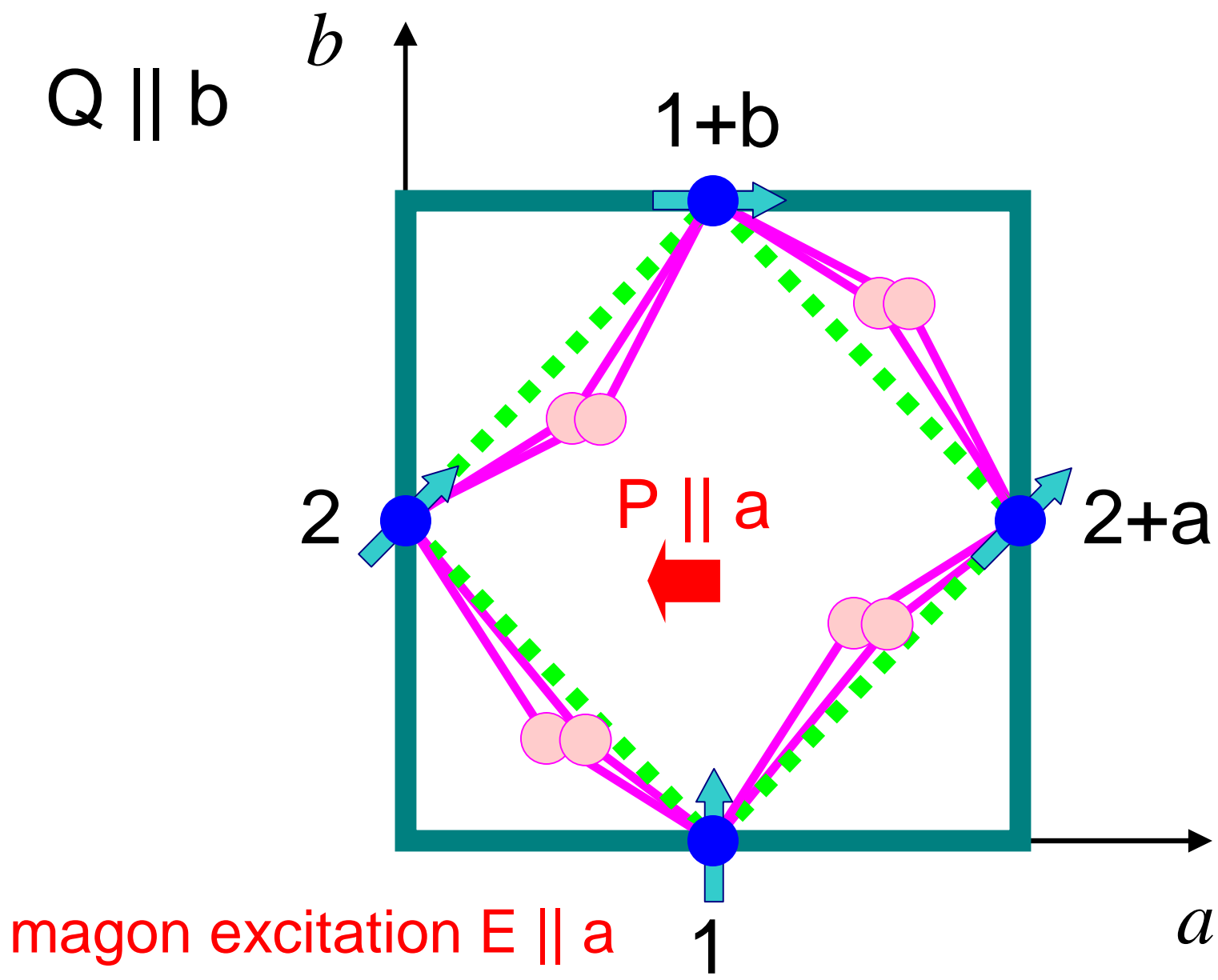
A. Pimenov et al  
*Nature Mat.* (2006)

YMn<sub>2</sub>O<sub>5</sub> & TbMn<sub>2</sub>O<sub>5</sub>

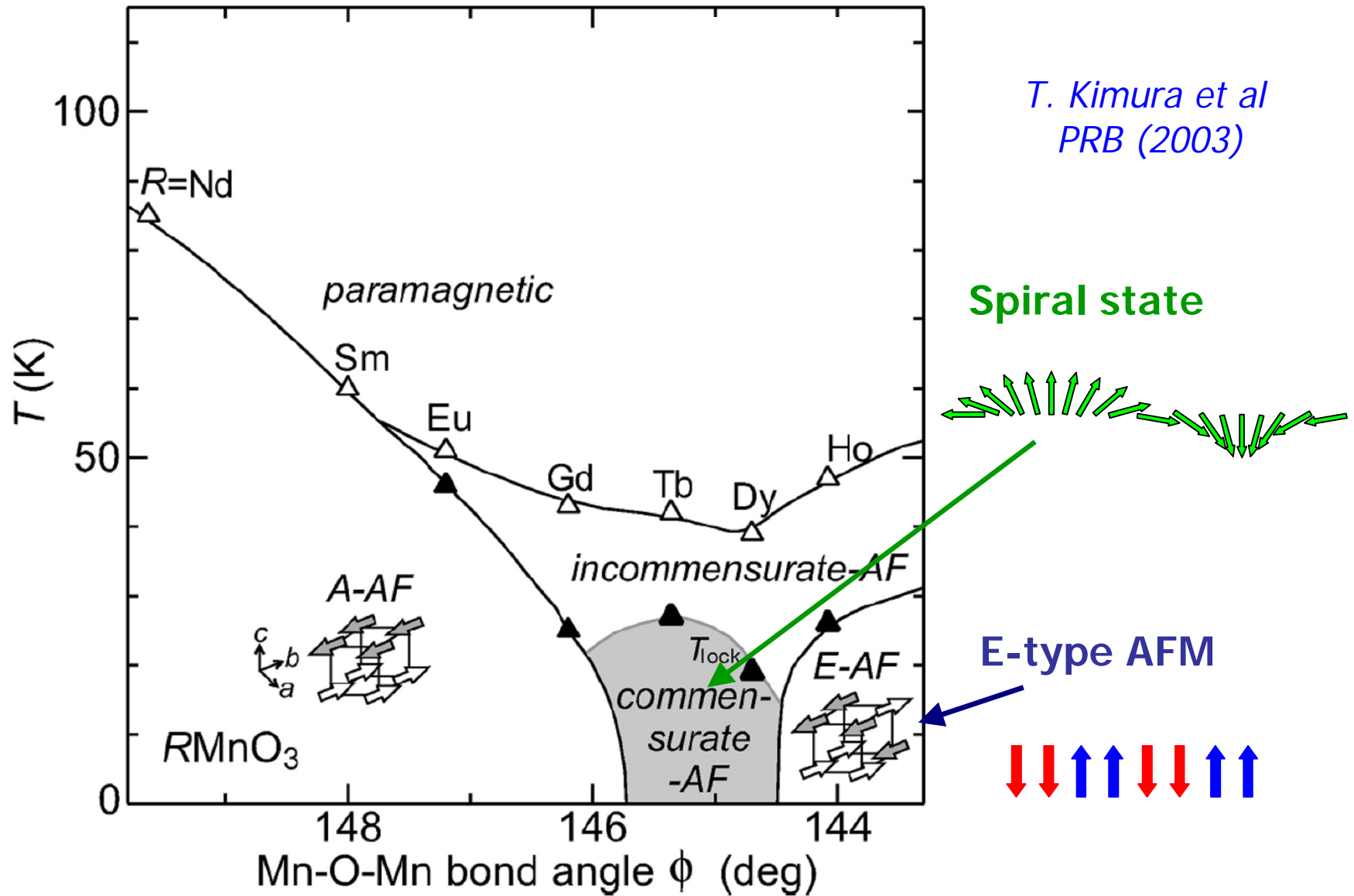


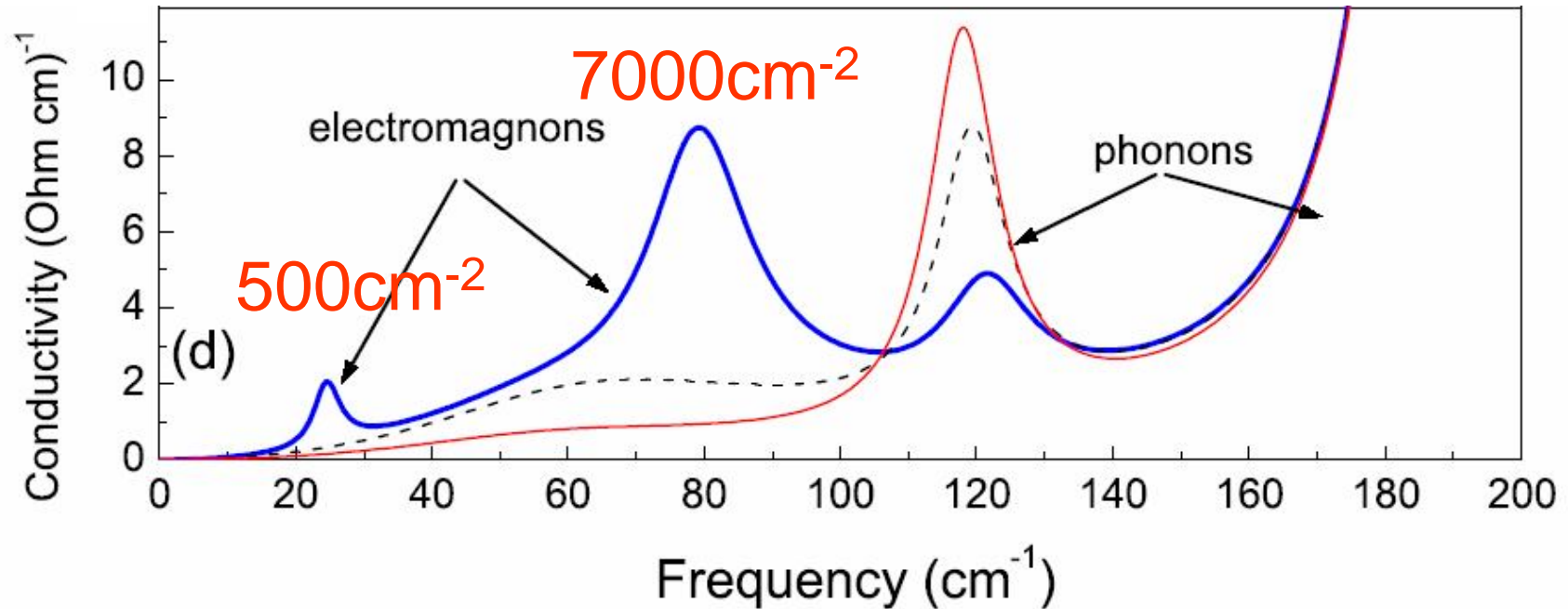
A.B. Sushkov et al (2006)

# RMnO<sub>3</sub>



# Orthorhombic $\text{RMnO}_3$





From the spectral weight of the giant electromagnon peak  $S \sim 7000 \text{ cm}^{-2}$  we obtain the polarization in the E-phase  $P \sim 1 \mu\text{C}\cdot\text{cm}^{-2}$

*R. Valdés-Aguilar et al, PRL 102 (2009)*

# Paramagnetism of ferroelectrics

*I. E. Dzyaloshinskii & D. E. Mills (2009)*

**Interaction of magnetic field with the soft ferroelectric mode**

$$\mathbf{H} \cdot \left[ \mathbf{P} \times \frac{\partial \mathbf{P}}{\partial t} \right]$$

$$H \parallel \langle P_z \rangle \neq 0 \quad \delta P_x \pm i \delta P_y \propto L_z$$

$$H_{\text{int}} = -\mu H_z L_z$$

# Dynamic magnetoelectric coupling

*MM, K. Nomura and N. Nagaosa, PRL **106**, 047204 (2011)*

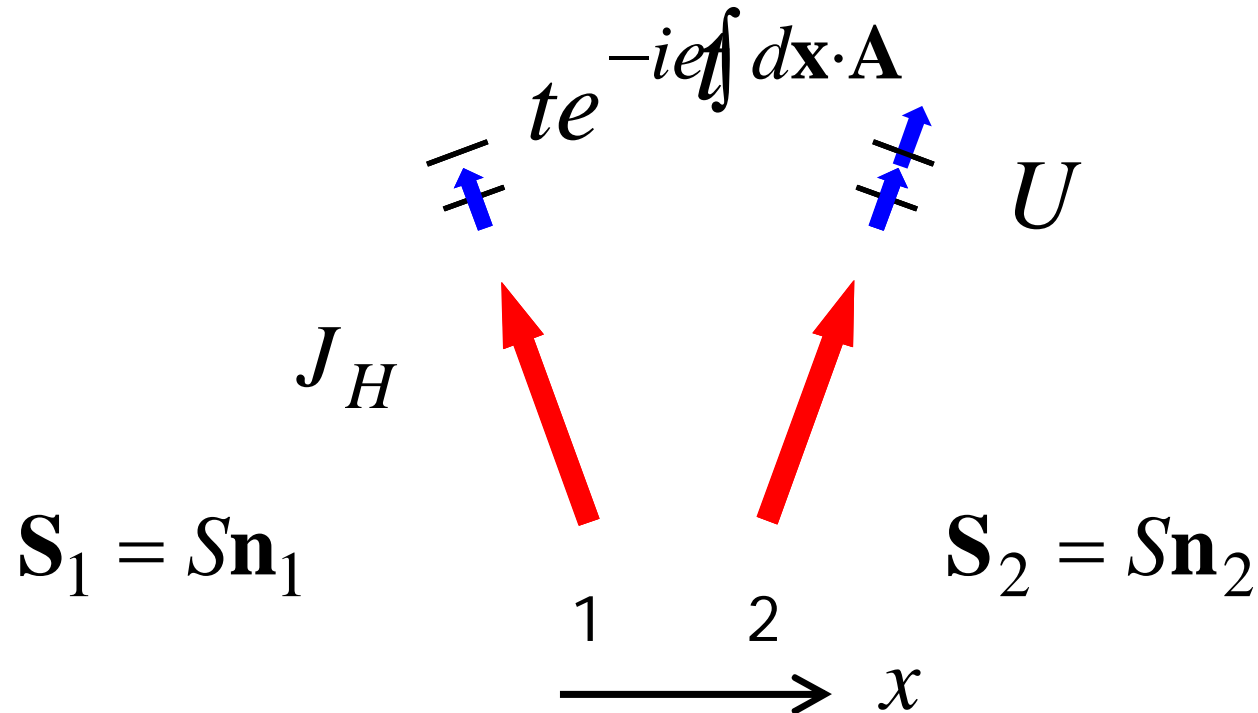
# Mechanisms of magnetoelectric coupling

Relativistic IDM mechanism  $\mathbf{P}_{12} \propto [\mathbf{r}_{12} \times [\mathbf{S}_1 \times \mathbf{S}_2]]$

Heisenberg exchange  $\mathbf{P}_{12} = \boldsymbol{\pi}_{12} (\mathbf{S}_1 \cdot \mathbf{S}_2)$

Dynamical ME coupling  $\mathbf{P}_{12} \propto \mathbf{r}_{12} ([\mathbf{S}_1 \times \mathbf{S}_2] \cdot (\dot{\mathbf{S}}_1 + \dot{\mathbf{S}}_2))$

# Exchange in multi-orbital Mott insulator



Dynamical ME coupling

$$\varepsilon [\mathbf{n}_1 \times \mathbf{n}_2] \cdot (\dot{\mathbf{n}}_1 + \dot{\mathbf{n}}_2)$$

$$\varepsilon = \left( \frac{t}{U} \right)^2 \frac{eaE_x}{U}$$

# Continuum limit

$$E_i (\mathbf{n} \cdot \partial_i \mathbf{n} \times \partial_t \mathbf{n}) \propto \mathbf{E} \cdot \mathbf{e}$$

Internal electric field



# Adiabatic motion

$$i\hbar \frac{\partial \psi}{\partial t} = \left( \frac{\mathbf{p}^2}{2m} - J\boldsymbol{\sigma} \cdot \mathbf{n} \right) \psi \quad \mathbf{n} = \mathbf{n}(\mathbf{x}, t)$$

Adiabatic projection

$$\psi = \chi |\mathbf{n}\rangle \quad \boldsymbol{\sigma} \cdot \mathbf{n} |\mathbf{n}\rangle = |\mathbf{n}\rangle$$

$$i\hbar \frac{\partial \chi}{\partial t} = \left( -ea_0 + \frac{\left( \mathbf{p} + \frac{e}{c} \mathbf{a} \right)^2}{2m} - J \right) \chi$$

Effective gauge potentials (*M. Berry*)

$$\begin{cases} a_0 = -\frac{\hbar}{2e} (1 - \cos \theta) \dot{\varphi}, \\ \mathbf{a} = \frac{\hbar c}{2e} (1 - \cos \theta) \nabla \varphi \end{cases}$$

# Effective gauge fields

$$\begin{cases} e_i = -\partial_i a_0 - \frac{1}{c} \dot{a}_i = \frac{\hbar}{2e} (\mathbf{n} \cdot \partial_i \mathbf{n} \times \dot{\mathbf{n}}), \\ h_i = [\nabla \times \mathbf{a}]_i = \frac{\hbar c}{2e} \delta_{iz} (\mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}) \end{cases}$$

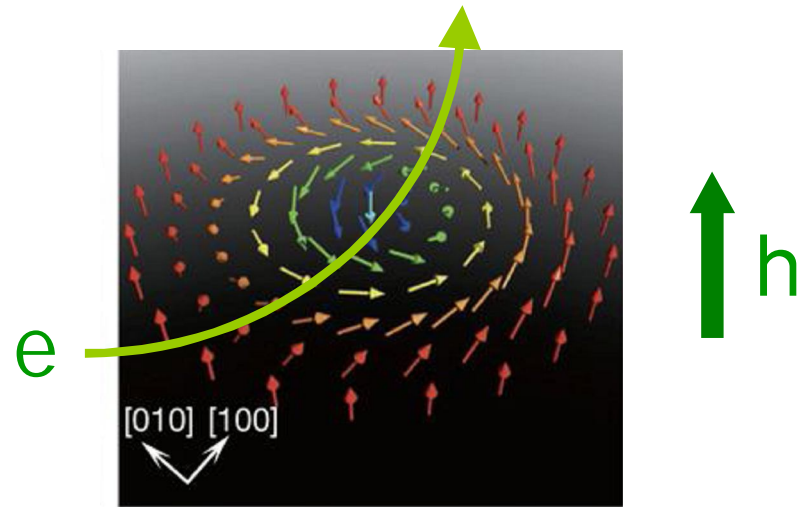
Force on spin-polarized electron

$$\Delta \mathbf{F} = -e \mathbf{e} - \frac{e}{c} [\mathbf{v} \times \mathbf{h}]$$

# Effects of internal gauge fields

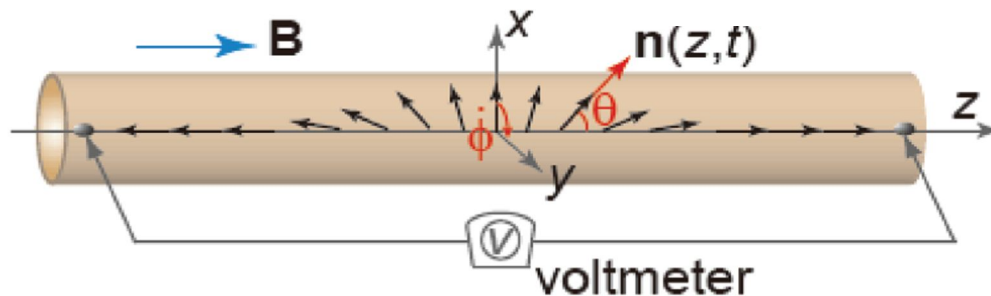
## Topological Hall effect

*A. Neubauer et. al. PRL (2009)*



## Spinmotive force:

$$\nabla \times \mathbf{e} = -\frac{1}{c} \frac{\partial \mathbf{h}}{\partial t}$$



*S. Barnes and S. Maekawa, PRL (2007)*

# Effects of dynamic coupling

$$L_{\text{int}} = -g \int d^3x \mathbf{E} \cdot \mathbf{e}$$

The meaning of  $\mathbf{e}$  in insulators is the electric polarization induced by time-dependent spins

Shift of spin texture in external electric field

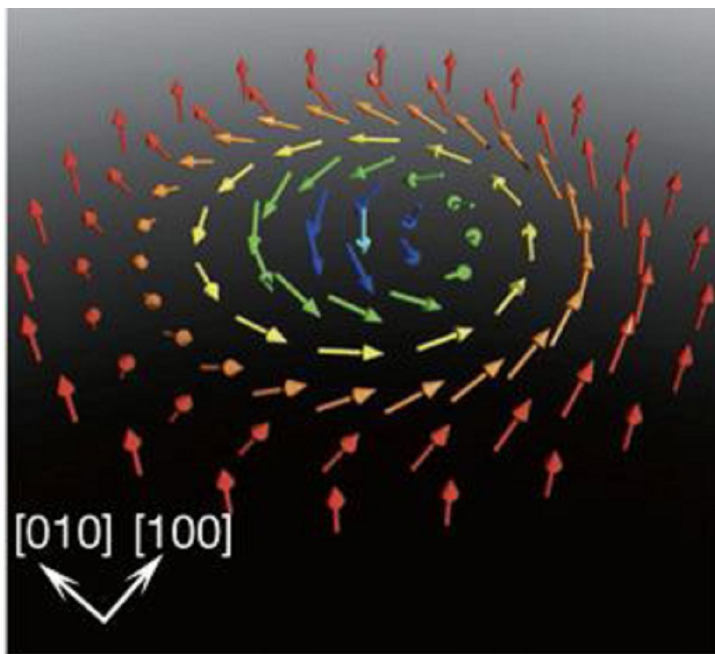
$$\mathbf{n}(\mathbf{x}, t) \rightarrow \mathbf{n}\left(\mathbf{x} + \frac{g}{S} \mathbf{E}, t\right)$$

tiny effect

# Topological textures

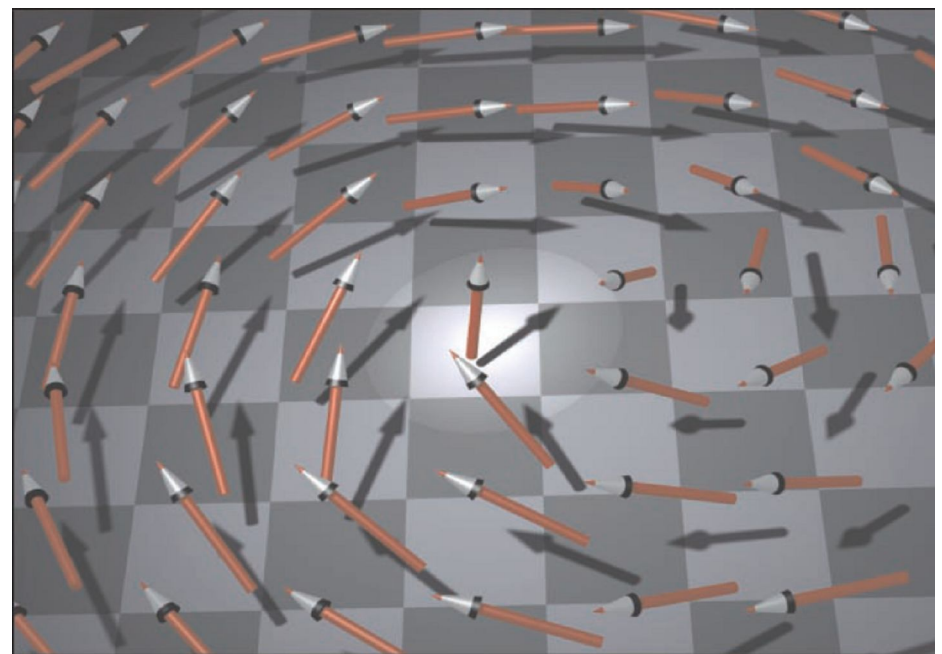
Topological charge:  $Q = \frac{1}{4\pi} \int d^2x (\mathbf{n} \cdot [\partial_x \mathbf{n} \times \partial_y \mathbf{n}])$

## Skyrmion



integer  $Q$

## Vortex



half-integer  $Q$

# Interaction with topological textures in thin films

Static skyrmion/vortex carries a flux of internal magnetic field

$$\Phi = \int d^2x h_z = Q\Phi_0 \quad \Phi_0 = \frac{hc}{e}$$

Moving Skyrmion/vortex induces internal electric field

$$\mathbf{e} = -\frac{1}{c} [\mathbf{V} \times \mathbf{h}]$$

*Jiadong Zhang, MM, Yung Hoon Han and Naoto Nagaosa, arXiv: 1102.5384*

$$\mathbf{n}(x, y, t) = \mathbf{n}(x - X(t), y - Y(t))$$

$$L_{\text{int}} = -gQ \int d^2x (E_x \dot{Y} - E_y \dot{X})$$

# Magnetic vortex in a disk

Lowest-energy model: Rotation of the vortex center

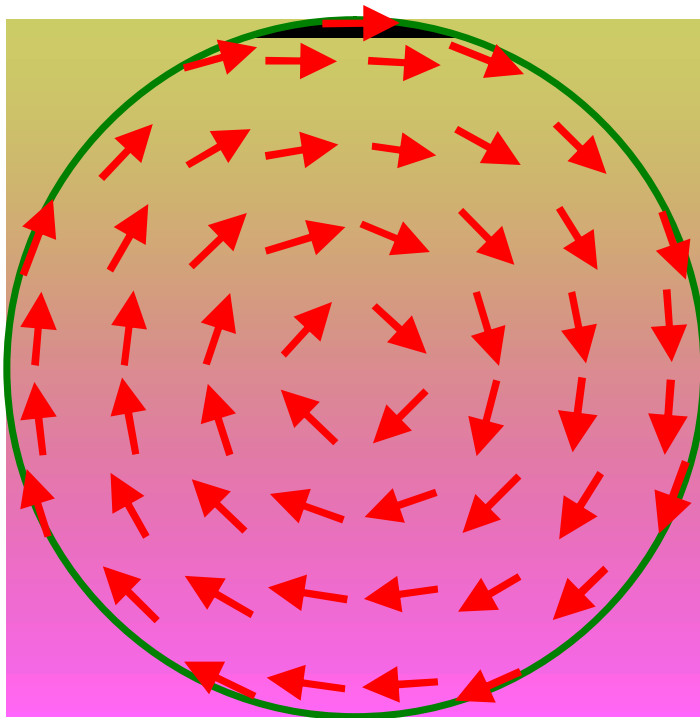
$$\mathbf{R}(t) = R(\cos \Omega t, \sigma \sin \Omega t)$$

Chirality:  $\sigma = \pm 1$

Excitation by rotating electric field

$$\mathbf{E}(t) = E(\cos \Omega t, \nu \sin \Omega t)$$

Circular polarization:  $\nu = \pm 1$



$$\frac{R}{gE} \approx \begin{cases} 1, & \nu = -\sigma \\ \frac{4\pi}{\alpha}, & \nu = \sigma \end{cases}$$

Gilbert damping



Resonant  
enhancement

# Solitonic array and Yukawa force in $\text{TbFeO}_3$

<http://arxiv.org/abs/1103.4275>

# Collaborators

## **Helmholtz Zentrum Berlin**

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Karel Prokes

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Niels Paduraru Jensen

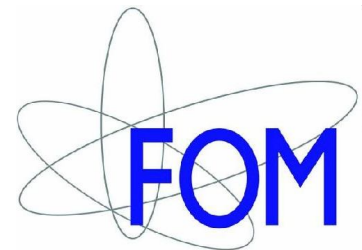
Luise Theil Kuhn

## **Niels Bohr Institute**

Kim Lefmann

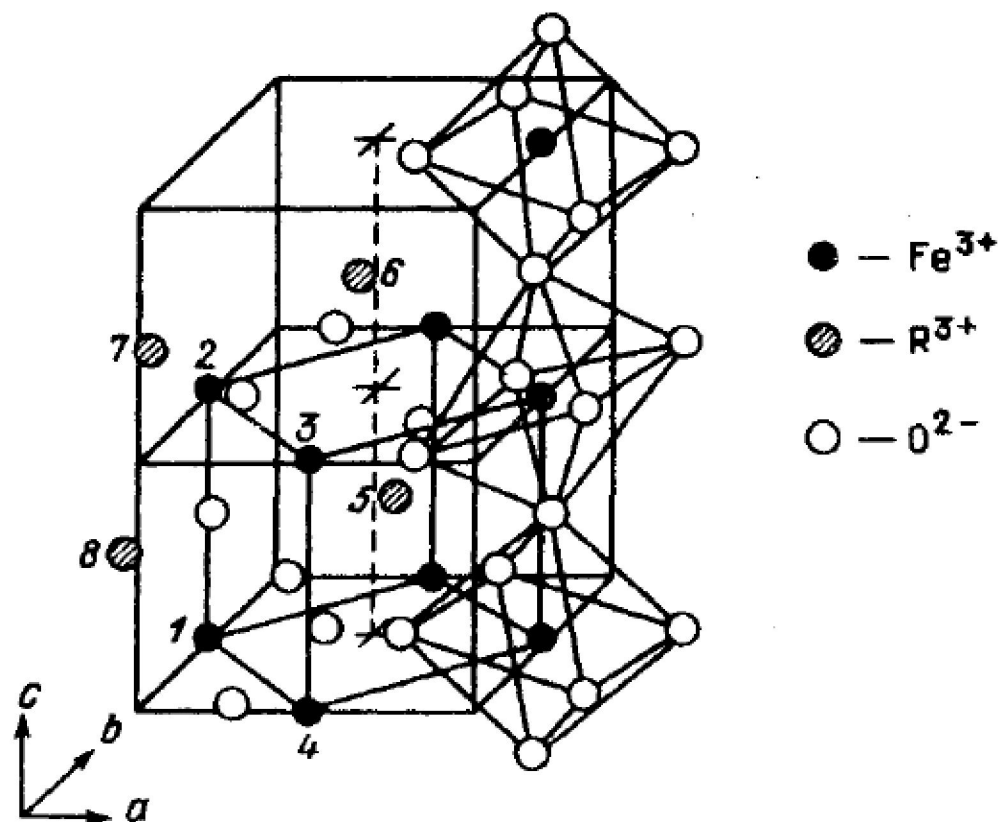
## **University of Groningen**

Sergey Artyukhin



# Orthoferrites

$\text{Fe}^{3+}$   $S=5/2$



1.  $T_{\text{Fe}} \sim 650\text{K}$  G-type AFM  $M \sim 0.1 \mu_{\text{B}}$
2.  $T_{\text{R}} \sim 10\text{-}100\text{K}$   $G_z \rightarrow G_x$   $G_z \rightarrow G_y$
3.  $T_{\text{RE}} < 5\text{K}$

# Representations of $Pbnm$

	Fe	RE	$\tilde{m}_x$	$\tilde{m}_y$	$m_z$
$\Gamma_1$	$A_x G_y C_z$	$C'_z$	+	+	+
$\Gamma_2$	$F_x C_y G_z$	$F'_x C'_y$	+	-	-
$\Gamma_3$	$C_x F_y A_z$	$C'_x F'_y$	-	+	-
$\Gamma_4$	$G_x A_y F_z$	$F'_z$	-	-	+
$\Gamma_5$		$G'_x A'_y$	-	-	-
$\Gamma_6$		$A'_z$	-	+	+
$\Gamma_7$		$G'_z$	+	-	+
$\Gamma_8$		$A'_x G'_y$	+	+	-

# Magnetic orders in GdFeO<sub>3</sub>

Fe<sup>3+</sup> S=5/2

T<sub>N</sub> = 661K



$$M_z \propto G_x$$

Gd<sup>3+</sup> S=7/2

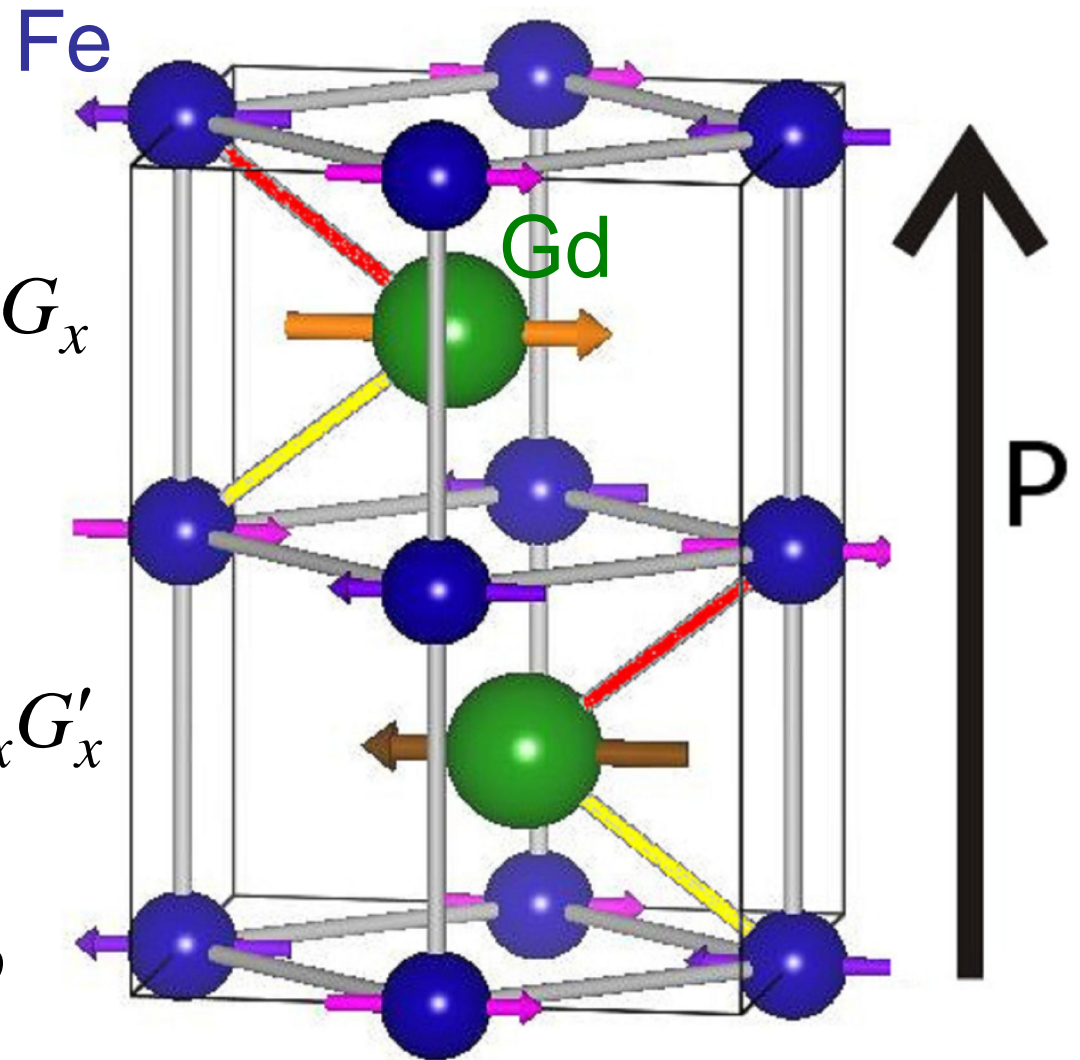
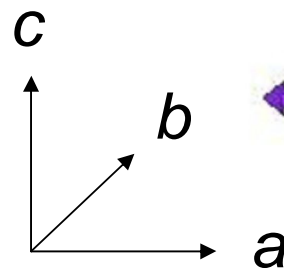
T'<sub>N</sub> = 2.5K



$$P_z \propto G_x G'_x$$

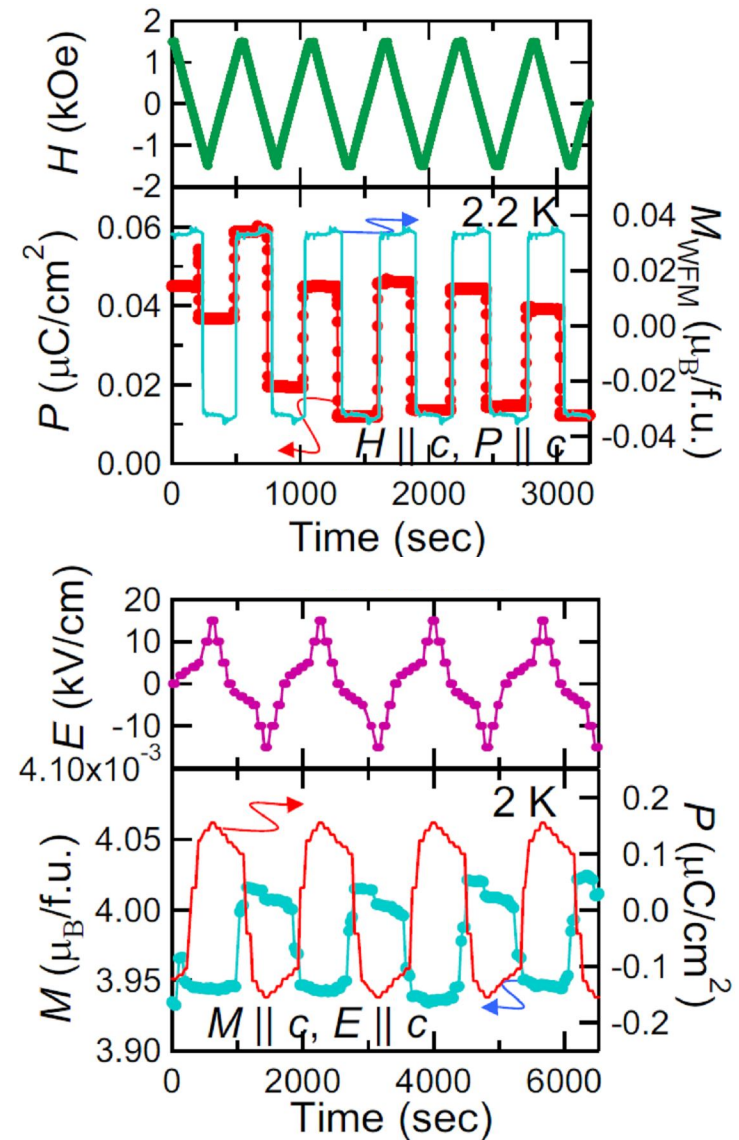
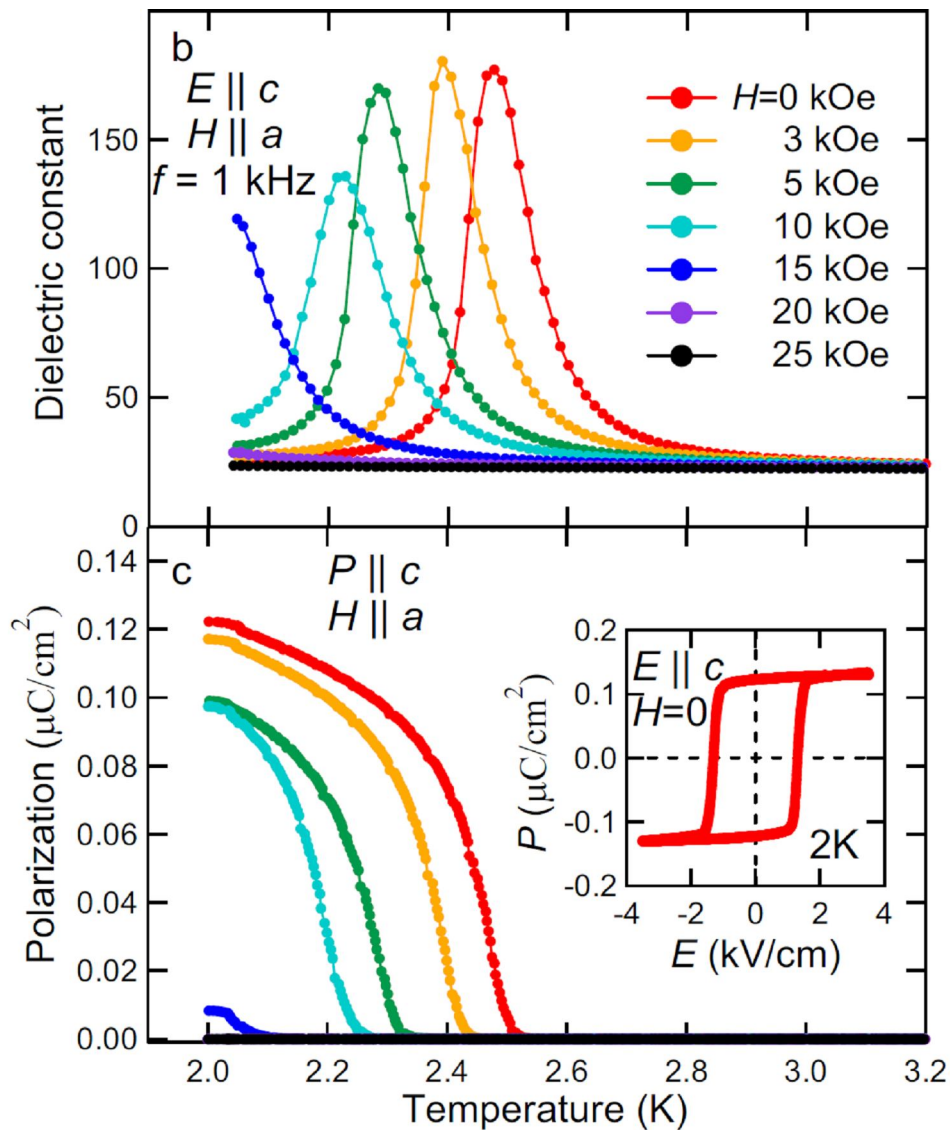
T < T'<sub>N</sub>

P<sub>z</sub> ~ 0.1 μC cm<sup>-2</sup>



# Magnetolectric effects in $\text{GdFeO}_3$

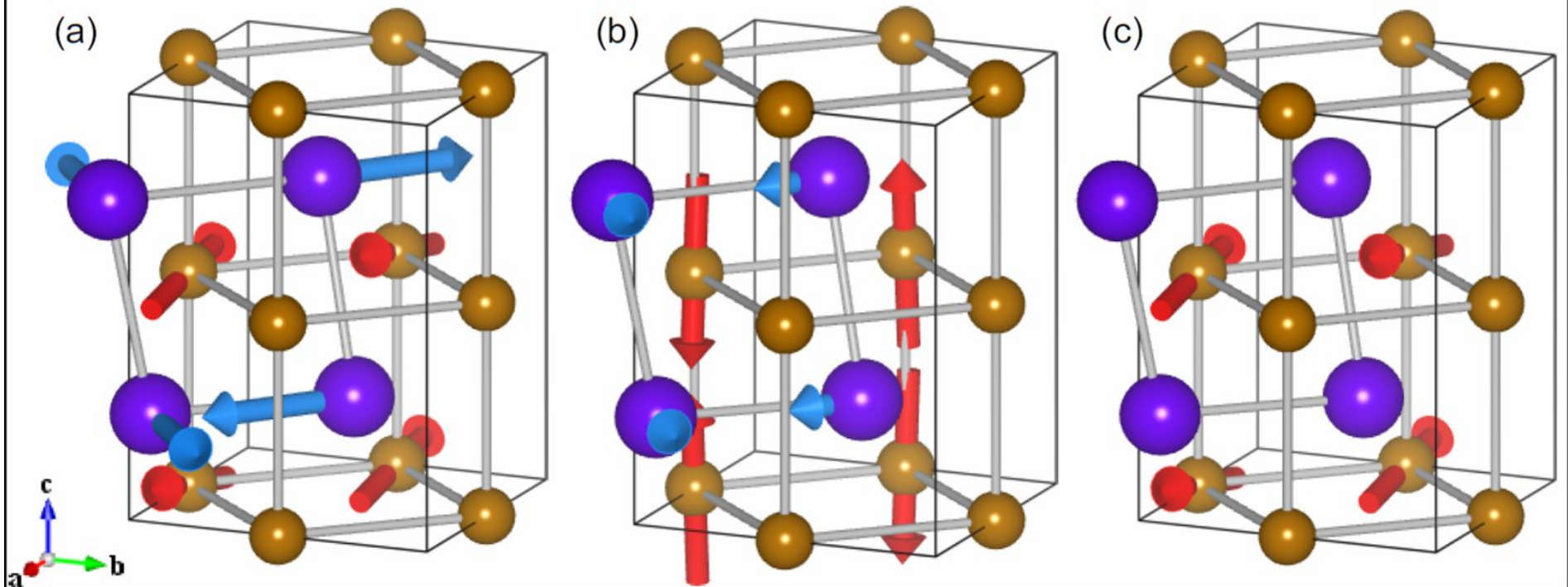
*Y. Tokunaga et al Nature Materials* **8**, 558 (2009)



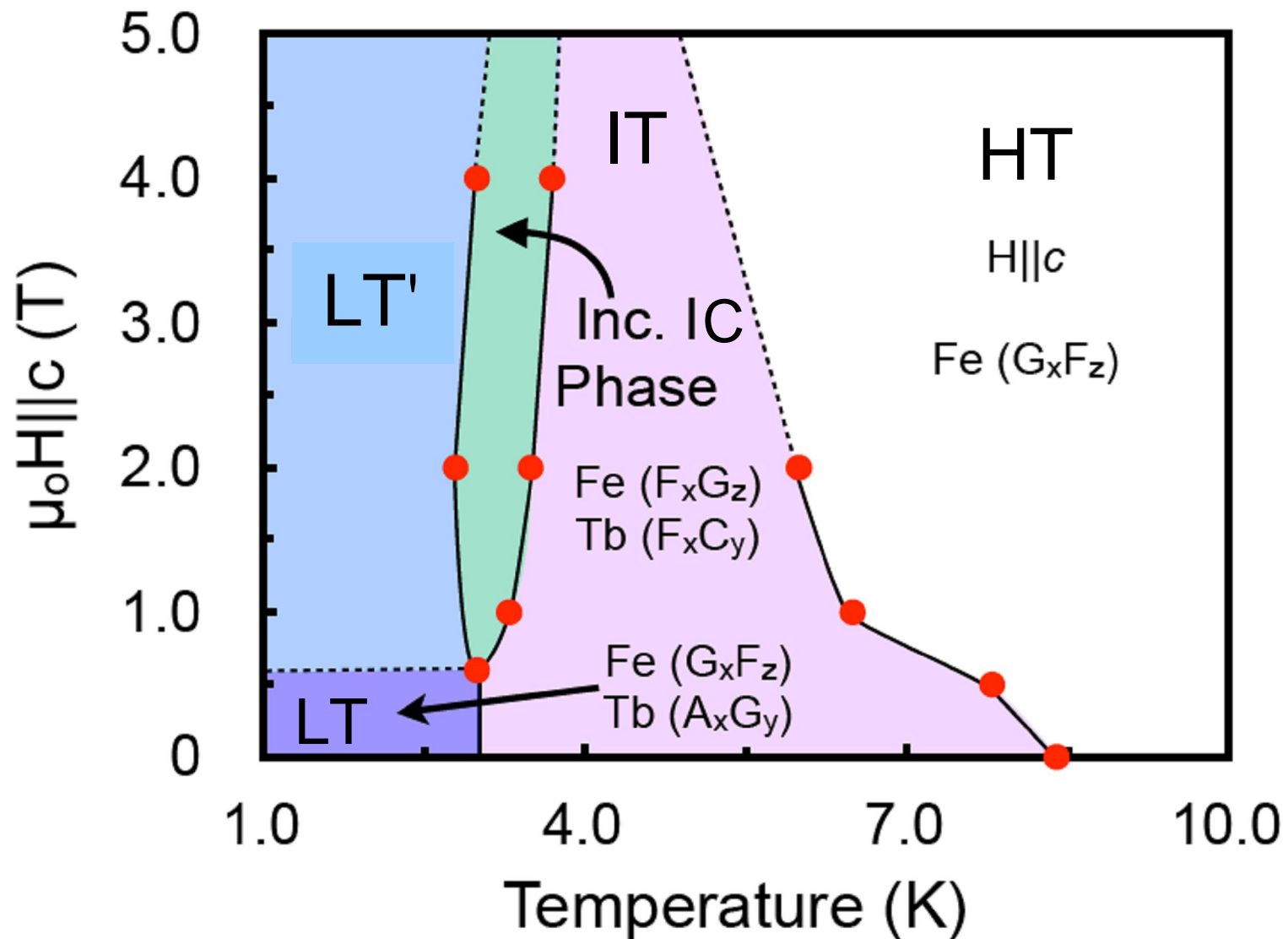
# TbFeO<sub>3</sub>

*E. F. Bertaut et al. (1967) K. P. Belov, A. K. Zvezdin, and A. A. Mukhin (1979)*

T(K)	T<3K	3K<T<8.5K	8.5K<T<650K
Fe	G <sub>x</sub> =5 μ <sub>B</sub> F <sub>z</sub> =0.15 μ <sub>B</sub> ξ <sub>1</sub> Γ <sub>4</sub>	G <sub>z</sub> =5 μ <sub>B</sub> F <sub>x</sub> =0.15 μ <sub>B</sub> ξ <sub>2</sub> Γ <sub>2</sub>	G <sub>x</sub> =5 μ <sub>B</sub> F <sub>z</sub> =0.15 μ <sub>B</sub> ξ <sub>1</sub> Γ <sub>4</sub>
Tb	A' <sub>x</sub> =6.5 μ <sub>B</sub> G' <sub>y</sub> =5 μ <sub>B</sub> η <sub>1</sub> Γ <sub>8</sub>	F' <sub>x</sub> =2.6 μ <sub>B</sub> C' <sub>y</sub> =1.8 μ <sub>B</sub> η <sub>2</sub> Γ <sub>2</sub>	

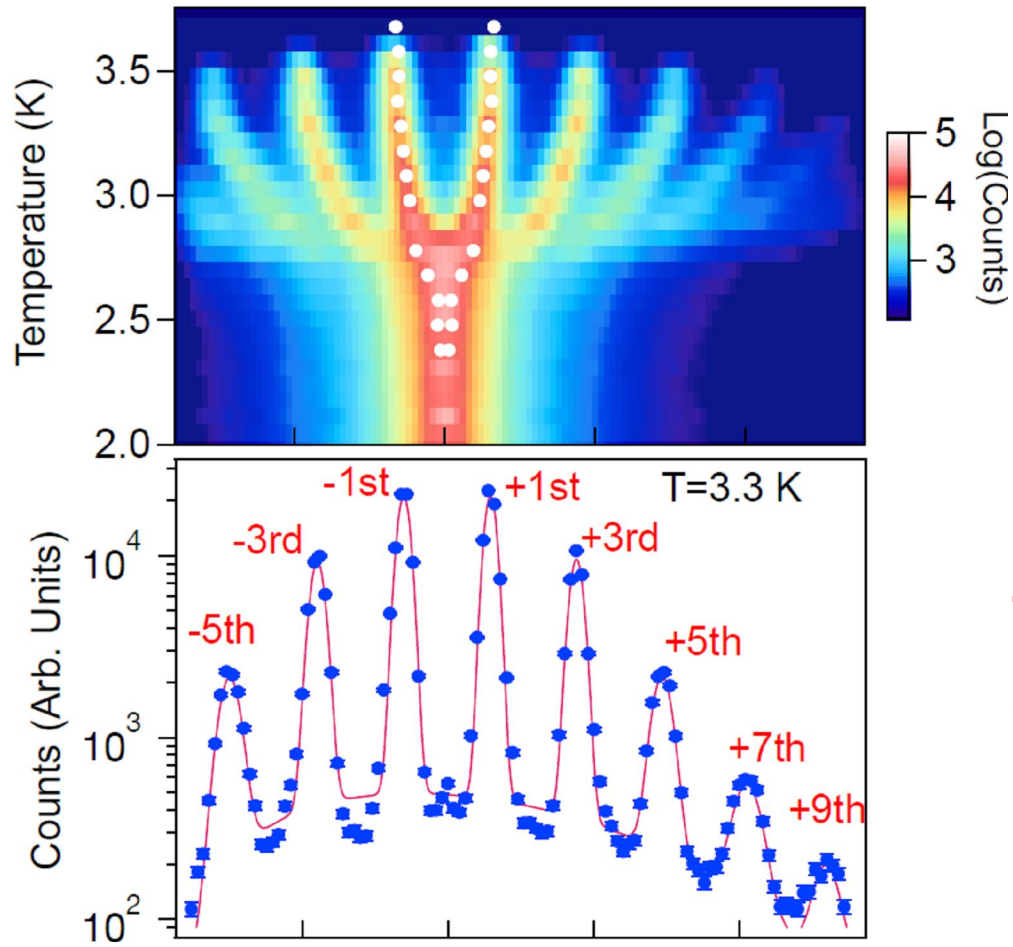


# Magnetic field phase diagram



# Incommensurate state

A-type reflection



**Sharp domain walls in  
Tb spin ordering  
separated by  $d = 170 \text{ \AA}$**

**Solitonic array**

# Landau theory

**Fe**  $f_{\text{Fe}} = \frac{c}{2} \left( \frac{d\theta}{dy} \right)^2 + \frac{K}{2} \sin^2 \theta - h \cos \theta$   $\Gamma_4 : \xi_1 = \cos \theta$   
 $\Gamma_2 : \xi_2 = \sin \theta$

**Tb**  $\Gamma_8 : \eta_1$   $\Gamma_2 : \eta_2$

$$f_{\text{Tb}} = \frac{c_1}{2} \left( \frac{d\eta_1}{dy} \right)^2 + \frac{c_2}{2} \left( \frac{d\eta_2}{dy} \right)^2 + \frac{a_1}{2} \eta_1^2 + \frac{a_2}{2} \eta_2^2 + \frac{b_1}{4} \eta_1^4 + \frac{b_{12}}{2} \eta_1^2 \eta_2^2 + \frac{b_2}{4} \eta_2^4$$

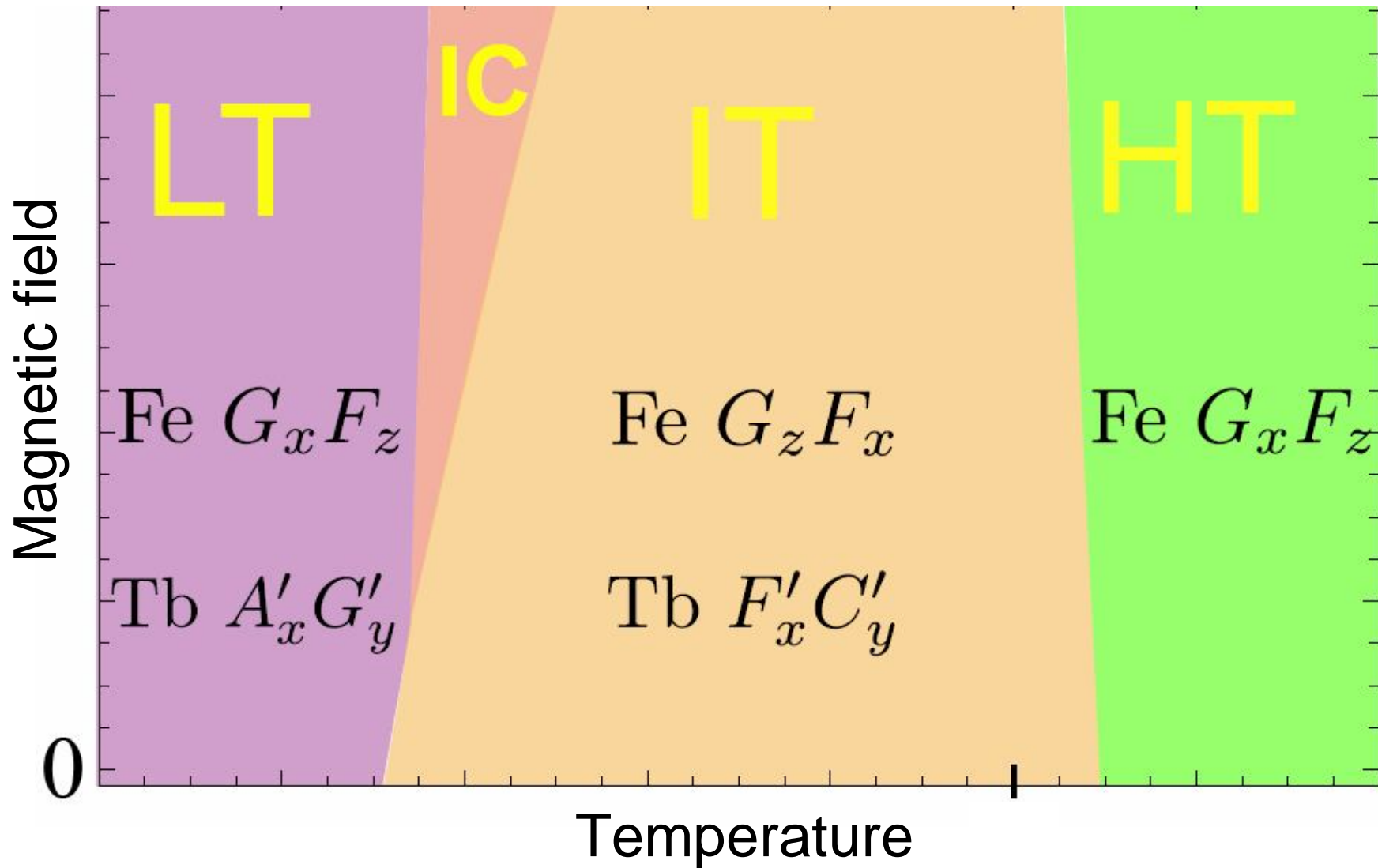
**Fe-Tb**

$$f_{\text{Fe-Tb}} = -\lambda \xi_2 \eta_2$$

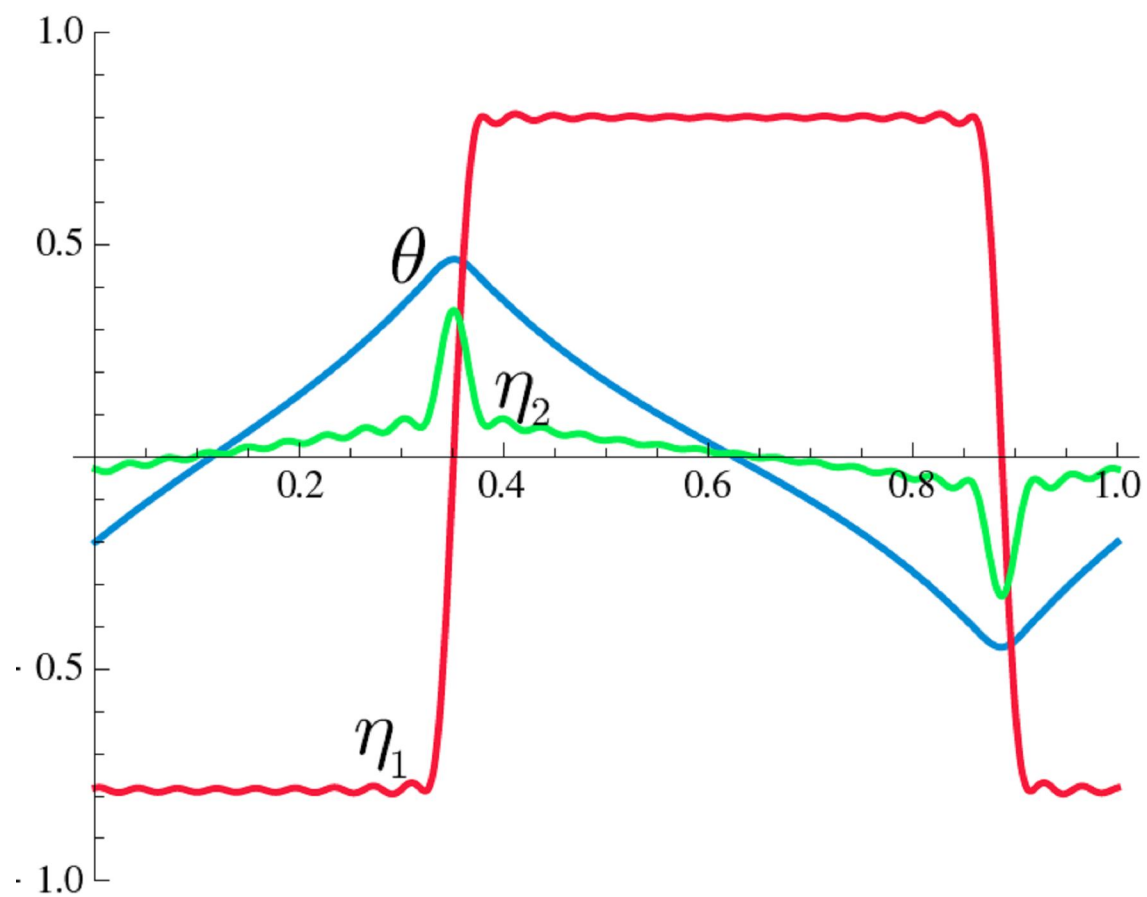
**Lifshitz invariant (cancelation of frustration)**

$$f_L = g_1 (\eta_1 \partial_y \xi_2 - \xi_2 \partial_y \eta_1) + g_2 (\eta_1 \partial_y \eta_2 - \eta_2 \partial_y \eta_1)$$

# Theoretical phase diagram



# Solitonic lattice

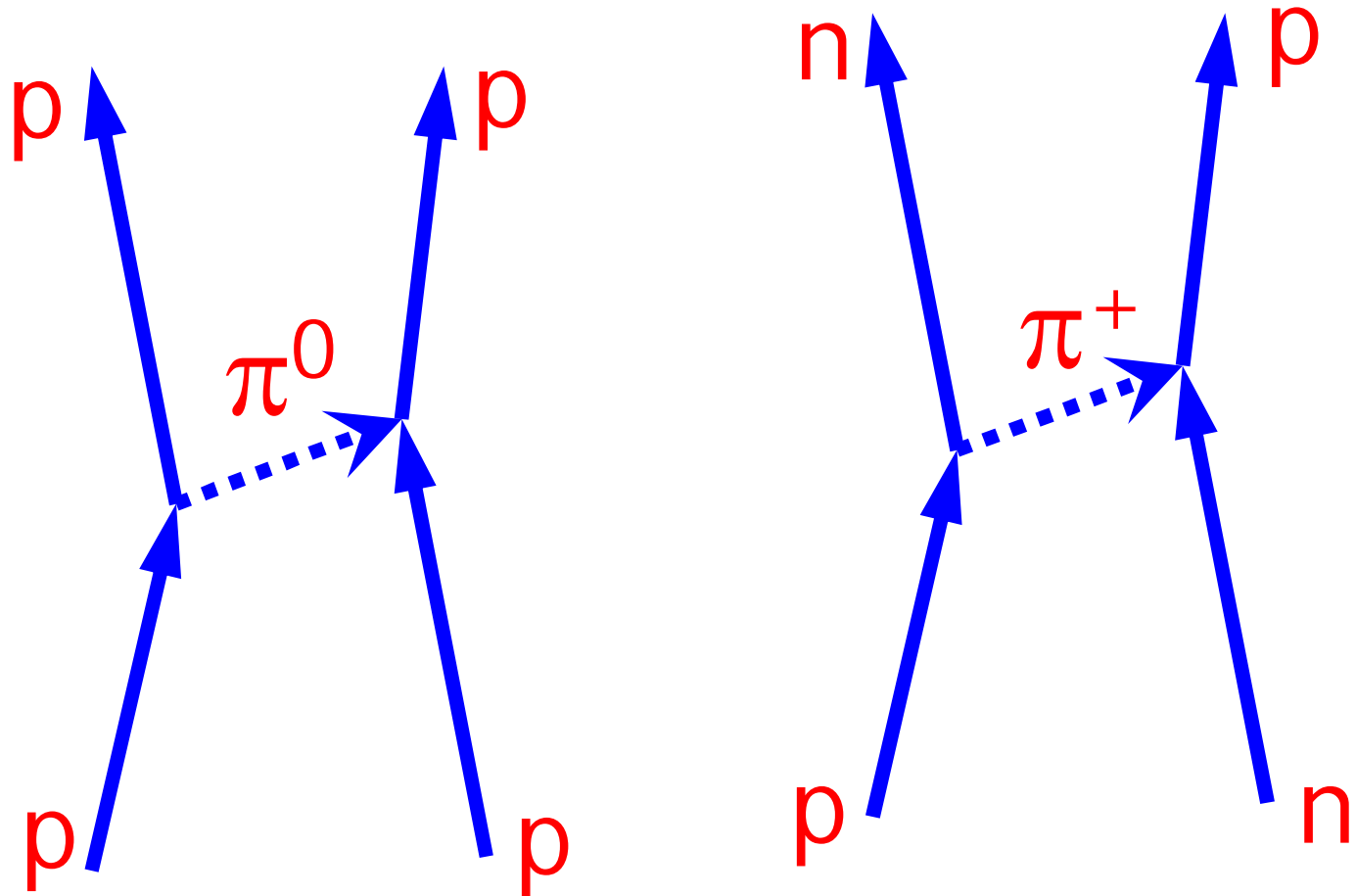




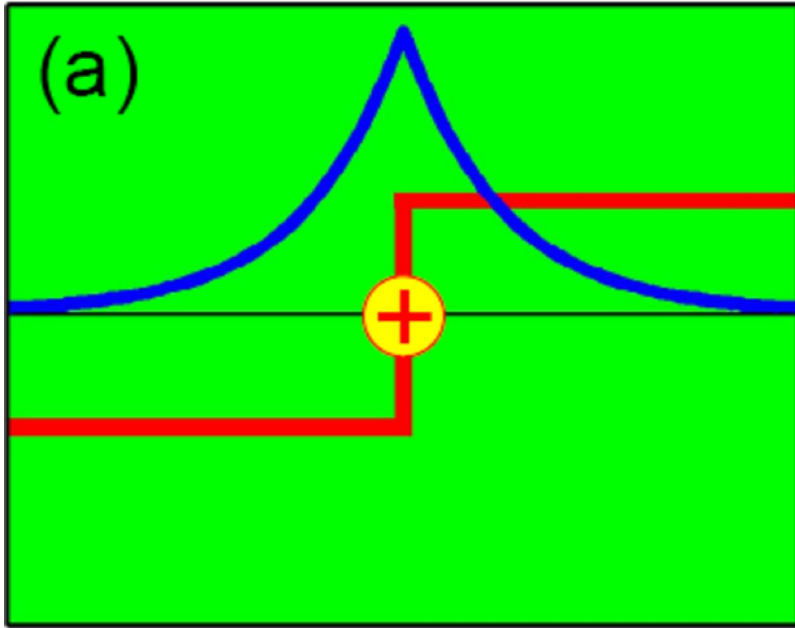
# Yukawa force

$$U(x) \propto e^{-\frac{x}{l}}$$

$$l = \frac{\hbar}{m_{\pi}c}$$



# Fe spin distortion around Tb domain wall



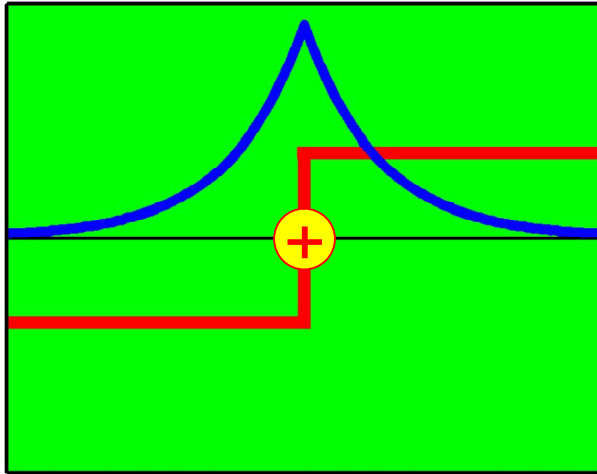
$$\theta(y) = \frac{Qg}{\sqrt{c(K+h)}} e^{-\frac{|y|}{l}}$$

$$l = m^{-1} = \sqrt{\frac{c}{K+h}}$$

$$F_{\theta} = -2g\theta(0)Q + \frac{1}{2} \int dy \left[ c \left( \frac{d\theta}{dy} \right)^2 + (K+h)\theta^2 \right]$$

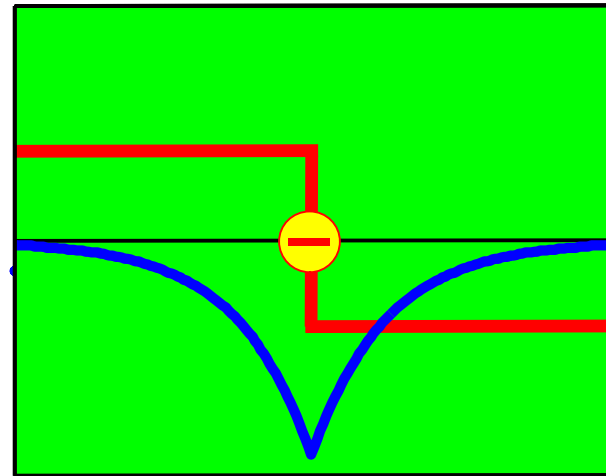
$$F_{DW} = F_{DW}^{(0)} - \frac{g^2}{\sqrt{c(K+h)}}$$

# Fe spin distortion around Tb domain wall



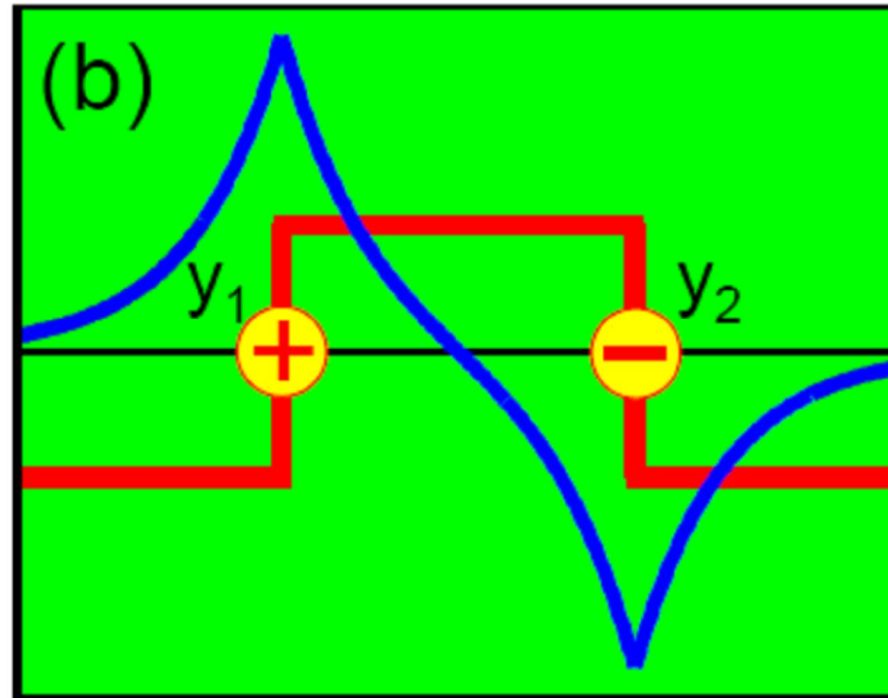
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$$Q = +1$$



$$Q = -1$$

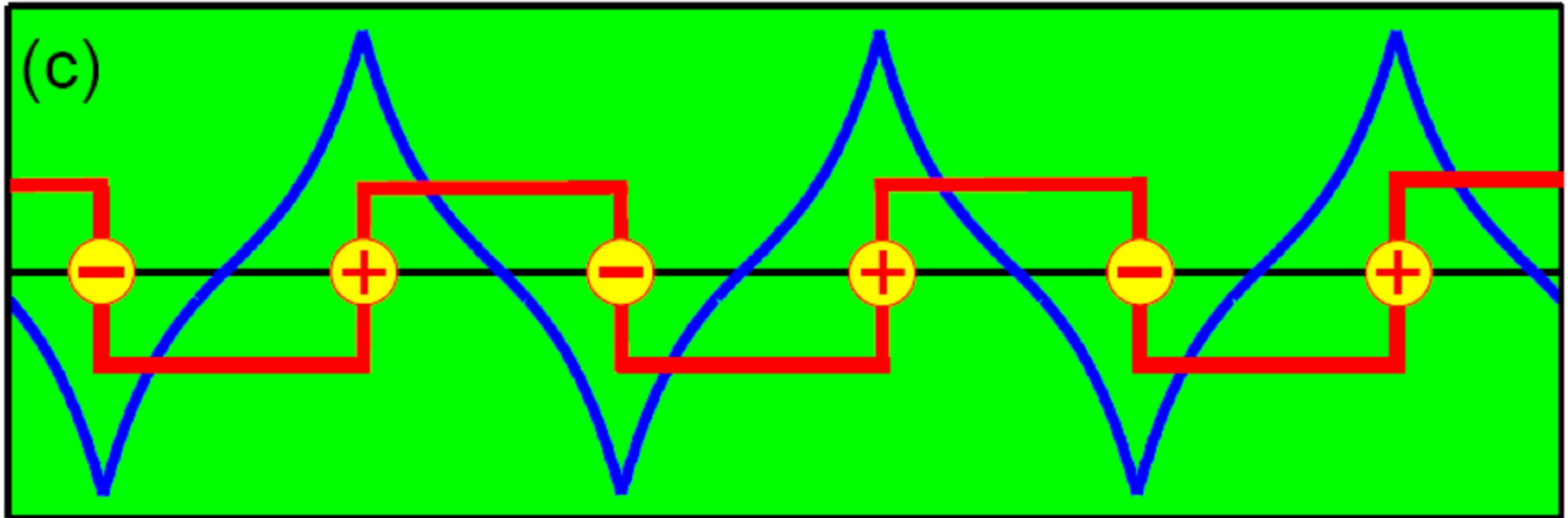
# Yukawa potential



Repulsion between neighboring domain walls

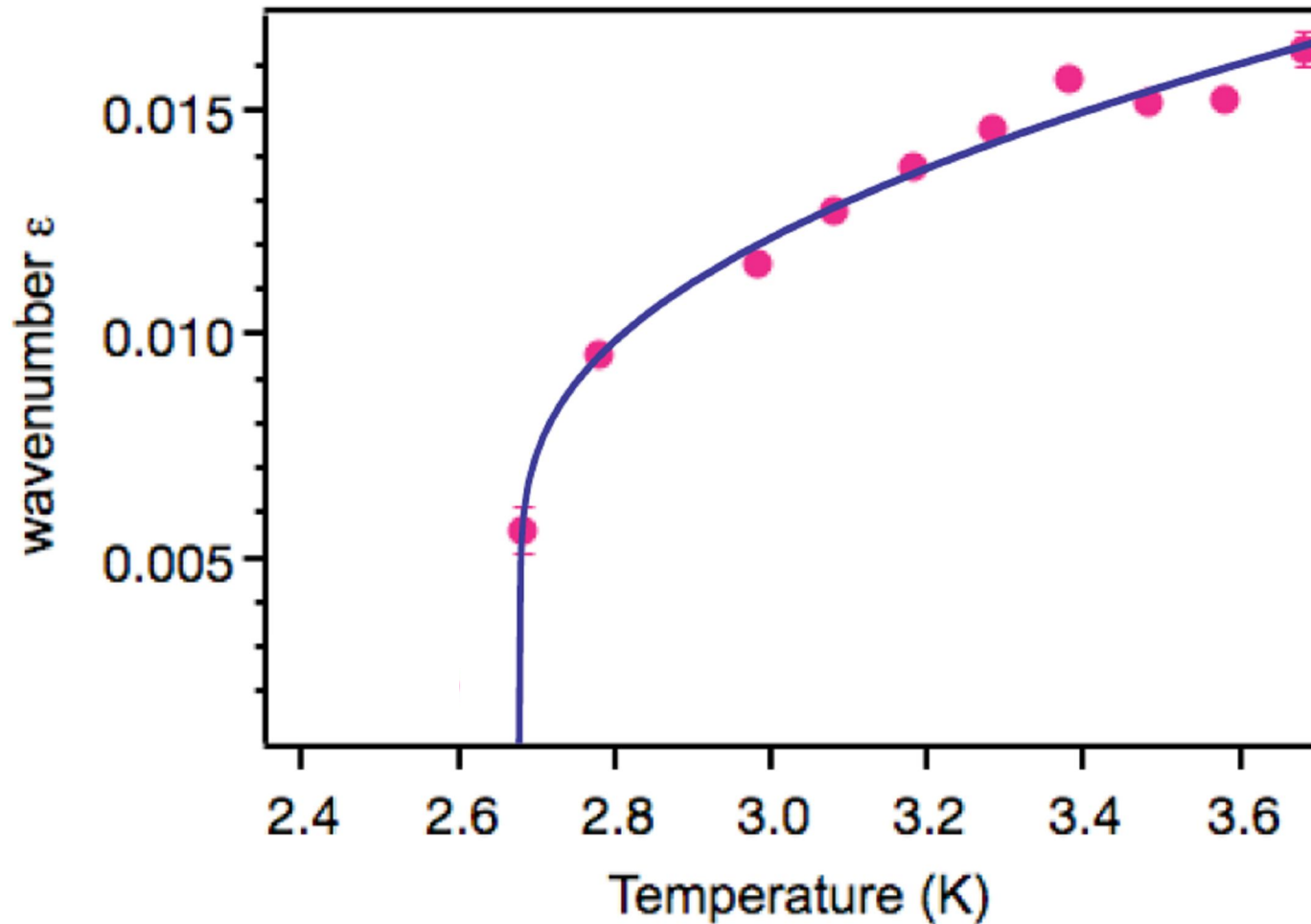
$$U(y_2 - y_1) = \frac{g^2}{\sqrt{c(K + h)}} e^{-\frac{|y_2 - y_1|}{l}}$$

# Solitonic array



$$F_{\theta} = - \sum_{n,m} Q_n U(y_n - y_m) Q_m$$

# T-dependence of wave vector

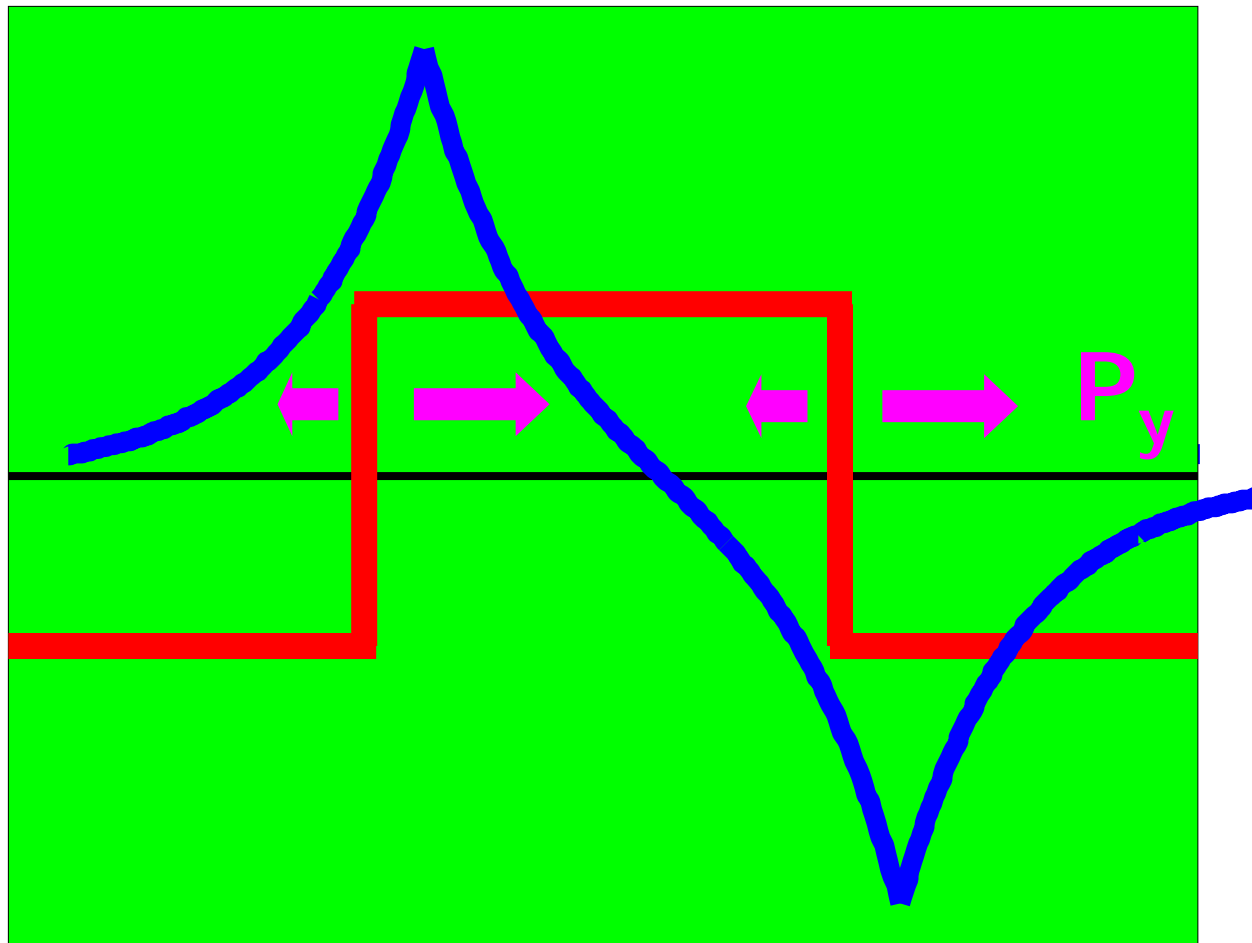


# Phason-electromagnon

$$\Gamma_8 \hat{\partial}_y \Gamma_2 - \Gamma_2 \hat{\partial}_y \Gamma_8$$

$$P_y \Gamma_8 \Gamma_2$$

$$E_y(t) \Gamma_8 \delta \Gamma_2(t)$$



# Conclusions

- Symmetry arguments make possible to predict new effects and understand the common origin of apparently different phenomena
- Advances in microscopic theory  
 $\alpha_{\parallel} = 10^{-4}$  for  $\text{Cr}_2\text{O}_3$  (*Scaramucci et al, 2010*)