Fermi liquid theory and ferromagnetism

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Landau Theoretical minimum 1966

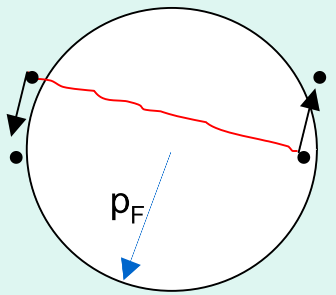
- Mathematics 1 — A.F.Andreev
- Classical mechanics — I.E.Dzyaloshinskii
- Statistical Physics — L.P.Gor’kov
- Quantum mechanics — I.E.Dzyaloshinskii
- Mathematics 2 — I.E.Dzyaloshinskii
- Field theory — I.A.Fomin
- Hydrodynamics and elasticity theory — V.L.Pokrovskii
- Electrodynamics of continuous media and kinetic theory of metals — E.M.Lifshitz
- Quantum electrodynamics — L.P.Pitaevskii

Outline

- Fermi liquid, polarized Fermi liquid, ferromagnetic Fermi liquid
- Spin waves in ferromagnetic Fermi liquid - Abrikosov & Dzyaloshinskii 1958
- Transverse spin diffusion in polarized Fermi liquid
- Dispersion with dissipation
- Quantum field theory approach - Dzyaloshinskii & Kondratenko 1976
- Quantum field theory approach - imaginary self energy part
- Conclusion: The intrinsic instability in a Fermi liquid with spontaneous magnetization

Fermi liquid

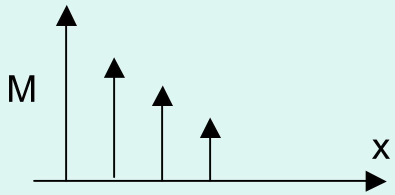
$T \ll \varepsilon_F$



Scattering rate $W = \frac{1}{2\tau} \propto T^2$ Landau & Pomeranchuk 1936

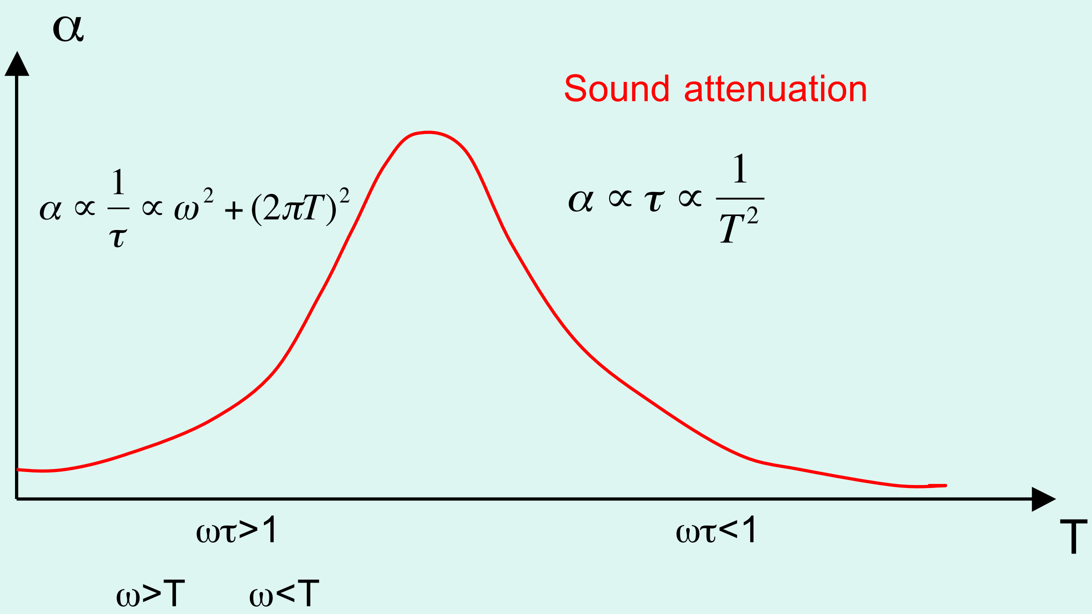
Viscosity $\eta \propto \frac{1}{T^2}$ Pomeranchuk 1950

Thermalconductivity $\kappa \propto \frac{1}{T}$ Abrikosov & Khalatnikov 1957



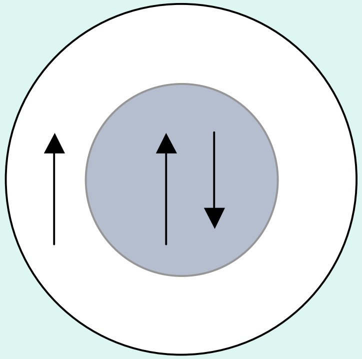
Longitudinal spin diffusion $D_{\parallel} \propto \frac{1}{T^2}$ D. Hone 1961

Sound attenuation Landau 1957



M.Meizel 2004

Magnetic polarization and the order parameter



$$\gamma \mathbf{H} = \gamma \mathbf{H}_0 - \hat{\mathbf{m}} {F_0}^a \int d\varepsilon \Delta n_0$$

$$\Delta n_0(\varepsilon_{\mathbf{k}}) = {n_0}^+ - {n_0}^-$$

$${n_0}^{\pm}(\varepsilon_{\mathbf{k}}) = n_0(\varepsilon_{\mathbf{k}} \mp \frac{\gamma H}{2}) = \frac{1}{\exp\left(\frac{\varepsilon_{\mathbf{k}} \mp \frac{\gamma H}{2} - \mu}{T}\right) + 1}$$

$$\hat{\mathbf{m}} = \mathbf{H}/H$$

Polarized Fermi liquid
Landau 1956

$$F_0^a > -1 \qquad \mathbf{H} = \frac{\mathbf{H_0}}{1 + {F_0}^a}$$

$$\mathbf{M} = \frac{\gamma^2 N_0}{4} \mathbf{H}$$

Ferromagnet
Fermi liquid
Stoner 1939

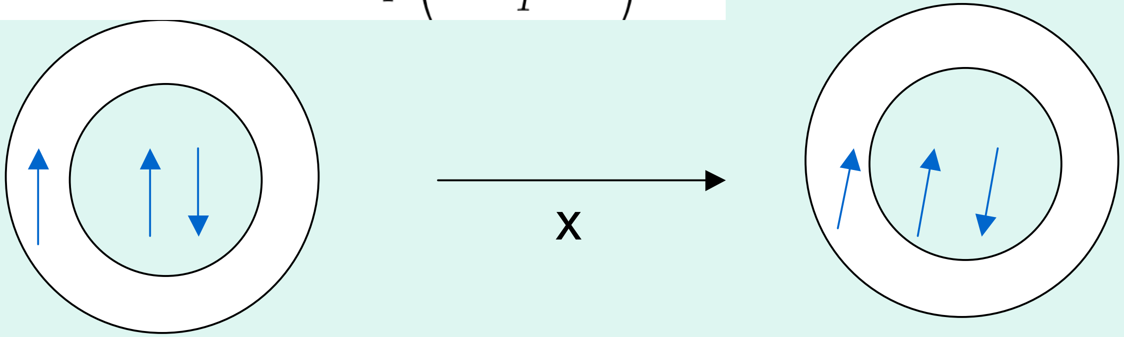
$$H_0 = 0 \qquad F_0^a < -1 \qquad \mathbf{H} = \hat{\mathbf{m}} \frac{4\varepsilon_F}{\gamma} \sqrt{6 \frac{1 + {F_0}^a}{{F_0}^a}}$$

$$\mathcal{F} = \alpha_0 (1 + F_0^a) M^2 + \beta M^4 \qquad T = 0$$

$$M = \sqrt{-\frac{\alpha_0(1 + F_o^a)}{2\beta}} \qquad F_0^a < -1$$

$$n_0^{\pm}(\varepsilon_{\mathbf{k}}) = n_0(\varepsilon_{\mathbf{k}} \mp \frac{\gamma H}{2}) = \frac{1}{\exp\left(\frac{\varepsilon_{\mathbf{k}} \mp \frac{\gamma H}{2} - \mu}{T}\right) + 1}$$

Spin waves



$$\hat{n}_{\mathbf{k}}(\mathbf{r},t) = n_{\mathbf{k}}(\mathbf{r},t) I + \boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t)\hat{\boldsymbol{\sigma}}$$

$$\hat{\varepsilon}_{\mathbf{k}}(\mathbf{r},t) = \varepsilon_{\mathbf{k}}(\mathbf{r},t)\hat{I} + \mathbf{h}_{\mathbf{k}}(\mathbf{r},t)\hat{\boldsymbol{\sigma}}$$

$$\hat{n}_{\mathbf{k}} = \hat{n}_{\mathbf{k}}{}^0 + \delta\hat{n}_{\mathbf{k}}$$

$$\hat{\varepsilon}_{\mathbf{k}} = \hat{\varepsilon}_{\mathbf{k}}{}^0 + \delta\hat{\varepsilon}_{\mathbf{k}}$$

$$\hat{n}_{\mathbf{k}}{}^0 = \bar{n}_0(\varepsilon_{\mathbf{k}})\hat{I} + \frac{1}{2}\Delta n_0(\varepsilon_{\mathbf{k}})(\hat{\mathbf{m}}\hat{\boldsymbol{\sigma}}) \quad \bar{n}_0(\varepsilon_{\mathbf{k}}) = \frac{1}{2}(n_0{}^+ + n_0{}^-) \quad \Delta n_0(\varepsilon_{\mathbf{k}}) = n_0{}^+ - n_0{}^-$$

$$\hat{\varepsilon}_{\mathbf{k}}{}^0 = \varepsilon_{\mathbf{k}}\hat{I} - \frac{1}{2}\gamma(\mathbf{B}\hat{\boldsymbol{\sigma}}) = \varepsilon_{\mathbf{k}}\hat{I} - \frac{1}{2}\gamma(\mathbf{H}_0\hat{\boldsymbol{\sigma}}) + \frac{1}{2}Sp'\int d\tau' f_{\mathbf{k}\mathbf{k}'}{}^{\sigma\sigma'}\delta\hat{n}'_{\mathbf{k}'}{}^0$$

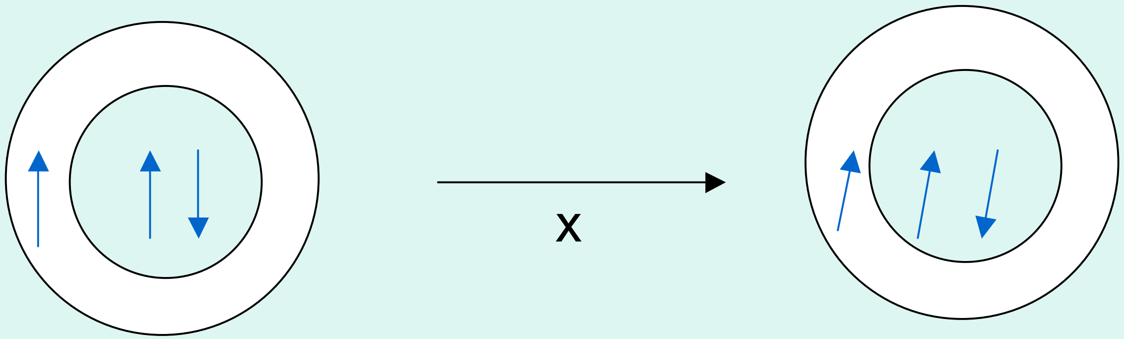
$$\frac{\partial \boldsymbol{\sigma}_{\mathbf{k}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{k}}}{\partial k_i}\frac{\partial \boldsymbol{\sigma}_{\mathbf{k}}}{\partial x_i} - \frac{\partial \mathbf{h}_{\mathbf{k}}}{\partial x_i}\frac{\partial n_{\mathbf{k}}}{\partial k_i} - 2(\mathbf{h}_{\mathbf{k}} \times \boldsymbol{\sigma}_{\mathbf{k}}) = \left(\frac{\partial \boldsymbol{\sigma}_{\mathbf{k}}}{\partial t}\right)_{coll}$$

Paramagnetic Fermi liquid under external field k=0, $\omega=\omega_L=\gamma H_0$ Silin 1957

Ferromagnetic Fermi liquid $H_0=0$, $T=0$, $\tau\propto 1/T^2=\infty$, $\omega \propto v_F{}^2 M k^2/\varepsilon_F{}^2$ Abrikosov & Dzyaloshinskii, 1959

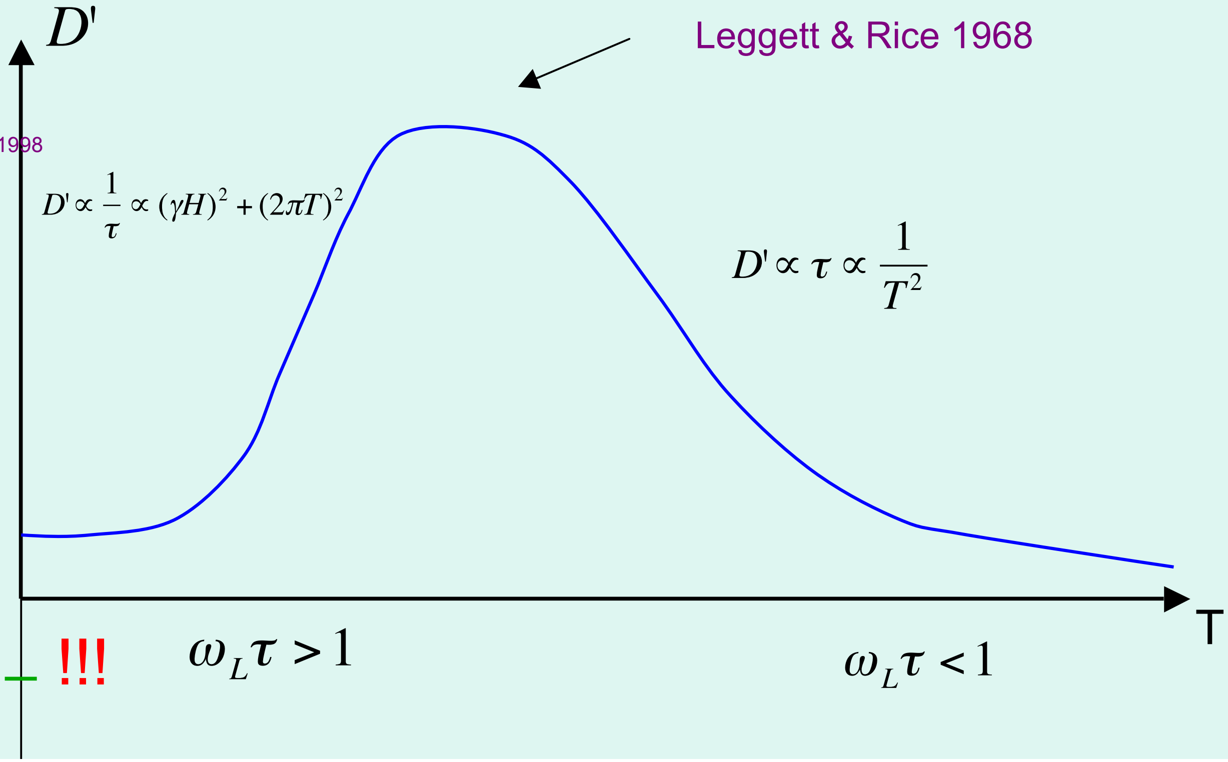
C.Herring, 1966 : "For a ferromagnetic metal…. If the spin of quasiparticle at Fermi surface is reversed, the corresponding quasiparticle state will no longer be closed to the Fermi surface, and it will have a finite, rather than an inifinitesimal, decay rate."

Transverse spin-diffusion in polarized liquid



$$\omega = \omega_L + (D'' - iD')k^2$$

Jeon&Mullin 1989,1992
Meyerovich&Musaelyan 1994
Golosov&Ruckenstein 1995, 1998
Mineev 2004



O.Buu et all 2002
H.Akimoto et al 2003
Vermeulen&Roni 2003
Vermeulen&Perisanu 2005

Stoner ferromagnet

Spin wave dispersion in Fermi liquid

V.Mineev 2005

$$\omega = \omega_L + (D'' - iD')k^2$$

$$\omega_L = \gamma H_0$$

$$D' \propto \frac{1}{\tau} = const((2\pi T)^2 + (\gamma H)^2)$$

$$\gamma H \tau >> 1$$

Polarized paramagnetic

$$D'' = \frac{v_F^2(1+F_o^a)(1+\frac{F_1^a}{3})}{3(F_0^a - \frac{F_1^a}{3})\gamma H} < 0$$

$$|D''| \propto \frac{1}{H}$$

$$D' = \frac{v_F^2(1+F_o^a))(1+\frac{F_1^a}{3})}{3(F_0^a - \frac{F_1^a}{3})^2(\gamma H)^2 \tau} > 0$$

$$D' \propto const$$

Ferromagnetic

$$D'' = \frac{v_F^2 F_0^a(1+\frac{F_1^a}{3})\gamma H}{3(4\varepsilon_F)^2(F_0^a - \frac{F_1^a}{3})} > 0$$

$$D'' \propto H$$

At $F_1^a=0$
It coincides with
derived by
T.Moriya 1985
From the Hubbard
model

$$D' = -\frac{v_F^2(F_0^a)^2(1+\frac{F_1^a}{3})}{3(4\varepsilon_F)^2(F_0^a - \frac{F_1^a}{3})^2 \tau} < 0$$

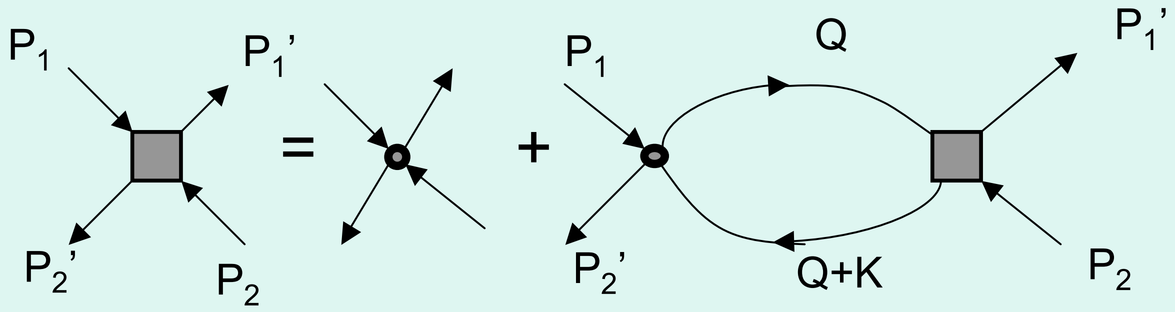
$$|D'| \propto H^2$$

Stoner liquid is unstable

$$\delta M \propto A \exp(ikx - i\omega t) \propto A \exp(ikx - iD''k^2 t - D'k^2 t)$$

Quantum field theory approach

P=(p,ε)



$$\Gamma(P_1,P_2,K)=\Gamma_1(P_1,P_2)-\frac{i}{(2\pi)^4}\int\Gamma_1(P_1,Q)G_+(Q)G_-(Q+K)\Gamma(Q,P_2,K)d^4Q$$

$$G_\pm(\mathbf{p},\varepsilon)=\frac{a_\pm}{\varepsilon-\varepsilon(\mathbf{p})\pm\gamma H/2+\mu+i\delta sign\varepsilon}$$

Dzyaloshinskii & Kondratenko 1976

$$G_\pm(\mathbf{p},\varepsilon)=\frac{a_\pm}{\varepsilon-\varepsilon(\mathbf{p})\pm\gamma H/2+\mu-ibv_F{}^2(p-p_\pm)|p-p_\pm|+i\delta sign\varepsilon}$$

$$b\propto m^*\sigma$$

$$G_+(\mathbf{q},\varepsilon)G_-(\mathbf{q}+\mathbf{k},\varepsilon+\omega)=\frac{2\pi i a^2}{v_F}\delta(\varepsilon)\delta(|\mathbf{q}|-p_0)\frac{\omega}{\omega-\gamma H-ib(\gamma H)^2/2-\mathbf{k}\mathbf{v}_F}+\tilde{\Phi}_{\rm reg}$$

$$\Gamma=\Gamma^{\mathbf{k}}+\Gamma^{\mathbf{k}}\tilde{\Phi}\Gamma$$

$$\Gamma^{\mathbf{k}}=\Gamma\left(\frac{\omega}{|\mathbf{k}|}\to 0\right)=(1+i\Gamma_1\tilde{\Phi}_{\rm reg})^{-1}\Gamma_1$$

$$\Gamma^{\mathbf{k}}\propto\ -\frac{1}{N_0(ck)^2}$$

Spin wave dispersion

$$\Gamma = \Gamma^{\mathbf{k}} + \Gamma^{\mathbf{k}}\tilde{\Phi}\Gamma$$

Polarized paramagnetic

$$\Gamma^{\mathbf{k}} \propto \frac{(\gamma H)^2}{N_0 v_F^2 k^2}$$

$$\omega \approx -\frac{v_F^2}{\gamma H}\left(1 + i\,\frac{b\gamma H}{2}\right)k^2$$

D'>0 D''<0

Ferromagnetic

$$\Gamma^{\mathbf{k}} \propto -\frac{p_F^2}{N_0 k^2}$$

$$\omega \approx \frac{\gamma H}{p_F^2}\left(1 + i\,\frac{b\gamma H}{2}\right)k^2$$

D' <0 D''>0

$$\omega = \omega_L + (D'' - iD')k^2$$

Conclusion

- In polarized Fermi liquid transverse spin waves dispersion law

$$\omega = \omega_L + (D'' - iD')k^2$$

derived by means kinetic equation and field theoretical methods is characterized by the finite attenuation at T=0.

- The spin wave spectrum in a ferromagnetic Fermi liquid proves to be unstable demonstrating difficulty of applying a Fermi liquid description to itinerant ferromagnetism.
- In ferromagnetic metals the two-moment approximation does not work. The quasiparticle interaction should be expanded not by the Legendre polynomials $f_{\mathbf{kk'}}{}^{\sigma\sigma'} = f_{\mathbf{kk'}}{}^{s}\hat{I}\hat{I}' + [f_0{}^{a} + f_1{}^{a}(\hat{\mathbf{k}}\hat{\mathbf{k}}')]\hat{\boldsymbol{\sigma}}\hat{\boldsymbol{\sigma}}'$ but by the eigen functions of the irreducible representation of the point crystal symmetry group taking into account spin-orbital interaction.

  Example: crystal with cylindrical (hexagonal) symmetry:

$$f_{\mathbf{kk'}}{}^{\sigma\sigma'} = f_{\perp}^{a}(\sigma_x k_x + \sigma_y k_y)(\sigma'_x k'_x + \sigma'_y k'_y) + f_{\parallel}^{a}(\sigma_z k_z)(\sigma'_z k'_z) + \ldots$$